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RESEARCH PROBLEMS

EDITED BY RICHARD GUY

In this Department the Monthly presents easily stated research problems dealing with notions ordinarily encountered in undergraduate mathematics. Each problem should be accompanied by relevant references (if any are known to the author) and by a brief description of known partial results. Manuscripts should be sent to Richard Guy, Department of Mathematics and Statistics, The University of Calgary, Calgary, Alberta, Canada, T2N 1N4.

IS THERE AN OCTIC RECIPROCITY LAW OF SCHOLZ TYPE?

DUNCAN A. BUELL AND KENNETH S. WILLIAMS

Recently there has been a revival of interest in the determination of rational reciprocity laws. The first law of this kind to be discovered was the famous Legendre-Gauss law of quadratic reciprocity

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{(p-1)(q-1)/4},$$

where p and q are odd primes and (p/q) is the Legendre symbol, which is plus or minus one according as p is or is not a quadratic residue of q.

If one assumes that $p \equiv q \equiv 1 \pmod 4$ and that $(p/q) = +1$, then the symbol $(p/q)_4$ is plus or minus one according as p is or is not a quartic residue of q. Under these assumptions Scholz's reciprocity law can be stated

$$\left(\frac{p}{q}\right)_4\left(\frac{q}{p}\right)_4 = \left(\frac{\varepsilon_p}{q}\right) = \left(\frac{\varepsilon_q}{p}\right), \tag{1}$$

where ε_p denotes the fundamental unit of the real quadratic field $\mathbf{Q}(\sqrt{p})$ and is an integer modulo q if p is a square modulo q. This law was proved by Scholz [5] in 1934 using class field theory. Since then, several more elementary proofs have been published [2], [3], [6], and the expository article by Emma Lehmer [4] gives an account of recent developments.

Our aim is to formulate an octic analogue of the Scholz reciprocity law under the assumption that

p and q are primes such that $p \equiv q \equiv 1 \pmod 8$, $(p/q)_4 = (q/p)_4 = +1$, so that the symbols $(p/q)_8$ and $(q/p)_8$ are defined. Then, by (1), we have $(\varepsilon_p/q) = (\varepsilon_q/p) = +1$, so that $(\varepsilon_p/q)_4$ is plus or minus one according as ε_p is or is not a quartic residue of q. When the norm of ε_{pq}, denoted by $N(\varepsilon_{pq})$, is equal to $+1$, we have to introduce the class number $h(pq)$ of the real field $\mathbf{Q}(\sqrt{pq})$. It is known [1] that if $p \equiv q \equiv 1 \pmod 8$, then

$$h(pq) \equiv \begin{cases} 0 \pmod 8, & \text{if } N(\varepsilon_{pq}) = -1, \\ 0 \pmod 4, & \text{if } N(\varepsilon_{pq}) = +1. \end{cases}$$

Armed with this information, we can state the following conjecture. Let p and q be primes such that $p \equiv q \equiv 1 \pmod 8$ and $(p/q)_4 = (q/p)_4 = +1$, then

$$\left(\frac{p}{q}\right)_8 \left(\frac{q}{p}\right)_8 = \begin{cases} \left(\frac{\varepsilon_p}{q}\right)_4 \left(\frac{\varepsilon_q}{p}\right)_4, & \text{if } N(\varepsilon_{pq}) = -1, \\ (-1)^{h(pq)/4} \left(\frac{\varepsilon_p}{q}\right)_4 \left(\frac{\varepsilon_q}{p}\right)_4, & \text{if } N(\varepsilon_{pq}) = +1. \end{cases}$$

This conjecture is based on numerical evidence alone and has been verified by machine computations for all 272 prime pairs (p,q) with $p < q < 2000$, $p \equiv q \equiv 1 \pmod 8$ and $(p/q)_4 = (q/p)_4 = +1$.

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CLASSROOM NOTES

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INDETERMINATE FORMS OF EXPONENTIAL TYPE

JOHN V. BAXLEY AND ELMER K. HAYASHI

When $\lim f(x)^{g(x)}$ yields an indeterminate of the form 0^0, ∞^0, or 1^∞, the usual procedure is to consider $g(x)\log f(x)$ and apply L'Hospital's Rule (see, for instance [1], [2], [4]). However, this method