ON NORRIE'S IDENTITY

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The first example expressing a biquadrate as the sum of four biquadrates was given by Norrie (University of St. Andrews 500th Anniversary Memorial vol., Edinburgh, 1911, 89). I give a simple demonstration of this result:

$$442^2 - 272^2 = 170\cdot 714 = 17^2\cdot 420,$$

hence $442^2-3\cdot 17^2=272^2+289\cdot 417=272^2+353^2-64^2$, but $3\cdot 17=2\cdot 26-1$, so

$$442^2 - 2\cdot 26\cdot 17 + 17 = 442^2 - 2\cdot 442 + 17 = 441^2 + 4^2 = 21^4 + 2^4 = 272^2 + 353^2 - 8^4.$$

Hence, $353^2+272^2=2^4+8^4+21^4$, but $353^2-272^2=81\cdot 625=15^4$, so $353^4=30^4+120^4+272^4+315^4$.

A FIBONACCI-LIKE SEQUENCE OF COMPOSITE NUMBERS

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Introduction. Let $S(L_0, L_1)=(L_0, L_1, L_2, \cdots)$ be a sequence of integers which satisfy the recurrence

$$L_{n+2} = L_{n+1} + L_n, \qquad n = 0, 1, 2, \cdots.$$

It is clear that the values of L_0 and L_1 determine $S(L_0, L_1)$, e.g., $S(0, 1)$ is just the sequence of Fibonacci numbers. It is not known whether or not infinitely many primes occur in $S(0, 1)$. On the other hand, if there is a prime p which divides both L_0 and L_1, then all the terms of $S(L_0, L_1)$ are divisible by p and in this case it is easily shown that only a finite number of the L_n can be prime. In this paper we exhibit two integers M and N with the following properties:

1. M and N are relatively prime.
2. No term of $S(M, N)$ is a prime number.

Preliminary remarks. Let L_0 and L_1 be arbitrary integers. Denote the nth Fibonacci number by F_n (where F_n is defined for *all* integers n by $F_0=0$, $F_1=1$, $F_{n+2}=F_{n+1}+F_n$, i.e., $F_{-1}=1$, $F_{-2}=-1$, etc.).

For any $m\geqq 0$ we have

$$L_m = 1\cdot L_m + 0\cdot L_{m+1} = F_{-1}L_m + F_0L_{m+1}$$

$$L_{m+1} = 0\cdot L_m + 1\cdot L_{m+1} = F_0L_m + F_1L_{m+1}$$

$$L_{m+2} = 1\cdot L_m + 1\cdot L_{m+1} = F_1L_m + F_2L_{m+1}.$$

Since $(F_nL_m+F_{n+1}L_{m+1})+(F_{n+1}L_m+F_{n+2}L_{m+1})=(F_{n+2}L_m+F_{n+3}L_{m+1})$, then by induction on n, it follows that

(1) $$L_{m+n} = F_{n-1}L_m + F_nL_{m+1}$$