EXERCISES 8, QUESTION 11

11. Let I be the ideal of $\mathbb{Z}+\mathbb{Z}\sqrt{-5}$ generated by $1+\sqrt{-5},\ 3+\sqrt{-5}$ and $19+9\sqrt{-5}$. Determine $\alpha, \beta \in \mathbb{Z}+\mathbb{Z}\sqrt{-5}$ such that $I =< \alpha, \beta >$.

Solution. As

$$19+9\sqrt{-5} = 4(1+\sqrt{-5}) + 5(3+\sqrt{-5})$$

we have

$$\begin{aligned} I &=< 1+\sqrt{-5}, 3+\sqrt{-5}, 19+9\sqrt{-5} > \\ &=< 1+\sqrt{-5}, 3+\sqrt{-5} > . \end{aligned}$$

■

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