CHAPTER 3, QUESTION 12

12. Let D be a unique factorization domain. Give an example to show that the following assertion is not true in general: If a is an irreducible element of D then $< a >$ is a maximal ideal of D.

Solution. Let $D = \mathbb{Z}[X]$. D is a unique factorization domain. The element X is an irreducible of D. Let $I =< 2, X >$. We showed in Question 5 that I is not principal so $I \neq< 1 >= D$. Also $2 \in I$ but $2 \notin< X >$. Thus the ideal $< X >$ is not maximal as

$$< X >\subset I \subset D. \qquad \blacksquare$$

June 20, 2004