Tutorial # 1, MATH 1104F, Winter 13
January 14, 2013

1. Determine which matrices are in reduced row echelon form (RREF) and which others are only in echelon form (REF)? (Note: A matrix is in RREF means it is in REF and each leading nonzero entry is 1 and all entries in the same column of leading 1 are zeros)

   (a) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (b) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

   (d) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

2. Given the following system of linear equations.

$$\begin{array}{rrrrl} x_1 & -7x_2 & & +6x_4 & = 5 \\ & & x_3 & -2x_4 & = -3 \\ -x_1 & +7x_2 & -4x_3 & +2x_4 & = 7 \end{array}$$

   Write down the augmented matrix of the above system and use elementary row operations to obtain the (reduced) row echelon form. Find the solutions in parametric form.

3. Determine the number of solutions to the following system.

   (a)

$$\begin{array}{rrl} x_1 & +3x_2 & = 2 \\ 3x_1 & +9x_2 & = 7 \end{array}$$

   (b)

$$\begin{array}{rrl} x_1 & +3x_2 & = 2 \\ 3x_1 & +9x_2 & = 6 \end{array}$$

   (c)

$$\begin{array}{rrl} x_1 & +3x_2 & = 2 \\ 3x_1 & +8x_2 & = 7 \end{array}$$

Solutions:

1. (a) RREF (b) REF (c) Neither RREF nor REF (d) Neither RREF nor REF

2. The associated augumented matrix is

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

The RREF of the associated augumented matrix is

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basic varibles are x_1 and x_3; free variable are x_2, x_4. Hence the solution set is $x_1 = 5 + 7s - 6t$, $x_2 = s$, $x_3 = -3 + 2t$, $x_4 = t$, s, t are any real numbers.

3. (a) The augumented matrix is

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix},$$

the system is inconsistent.

(b) The augumented matrix is

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix},$$

the system has infinitely many solutions.

(b) The augumented matrix is

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 8 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 1 \end{bmatrix},$$

the system has unique solution.