HOMEWORK No 9 (March 23, 2005)

Exercise 1. *Using the (right) translation of Cayley, express the symmetric group* $S(3)$ *as a subgroup of* $S(6)$.

Exercise 2. *Denote by* G *the group of the cross-ratios, i.e. the group of the real functions generated by* $f(x) = 1/x$ *and* $g(x) = 1 - x$ *with the operation of composition of functions (thus, e.g.* $(f \circ g)(x) = 1/(1-x)$ *and* $(g \circ f)(x) = (x-1)/x$*). Describe the group* G.

Exercise 3. *Show that the group of symmetries of the pentagon is isomorphic to the dihedral group* $D(10) = \langle a, b \mid a^5 = b^2 = (ab)^2 = 1 \rangle$.

Exercise 4. *Define the binary operation* $\star$ *on* $G = \{(a,b) \mid 0 \neq a \in \mathbf{R}, b \in \mathbf{R}\}$ *of pairs of real numbers by* $(a,b) \star (c,d) = (ac, bc + d)$. *Show that* G *is a group and* $H = \{(1,b) \mid b \in \mathbf{R}\}$ *is its normal subgroup, isomorphic to the additive group of real numbers. Describe the factor group* G/H.

Exercise 5. *Following the pattern of Exercise 4., define the group*

$$G = \{(a,b) \mid 0 \neq a \in \mathbf{Z_p}, b \in \mathbf{Z_p}\}$$

of order $p(p-1)$. *Describe its structure for*
(a) $p = 3$ *and*
(b) $p = 5$.

Exercise 6. *Prove (*G, H, K *are groups):*
(a) *If* $\phi : G \to H$ *and* $\psi : H \to K$ *are homomorphisms, then their composite* $\phi \circ \psi : G \to K$ *is a homomorphism.*
(b) *If* $\phi : G \to H$ *is a homomorphism and* K *is any subgroup of* G, *then* $\phi(K) = \{\phi(a) \mid a \in K\}$ *is a subgroup of* H.
(c) *The mapping* $\phi : G \to G$ *defined by* $\phi(a) = a^2$ *is a homomorphism (i.e. endomorphism) if and only if* G *is abelian.*

Solutions will be sent to all students by e-mail.
They will be also available in the display case opposite of my office 4205HP
on Monday, March 28, 2005.