HOMEWORK No 8 (March 16, 2005)

Exercise 1. *Show that every element of the permutation group* $S(n)$ *can be written as a product of transpositions* ($2-$*cycles). Write the permutation* $(1257)(32467)(517) \in S(7)$ *as a product of transpositions.*

Exercise 2. *Show that the group of symmetries of the square is isomorphic to the dihedral group* $D(8) = \langle a, b \mid a^4 = b^2 = (ab)^2 = 1 \rangle$.

Exercise 3. *Describe the left and the right cosets of the cyclic subgroup*
(i) $\langle ab \rangle$ *of* $D(8)$;
(ii) $\langle a^2 \rangle$ *of* $D(8)$.

Exercise 4. *Let* $\phi : \mathbf{Z_{12}} \to S(4)$ *be the homomorphism satisfying* $\phi(\bar{1}) = (1234)$. *Determine* $\phi(x)$ *for each element* $x \in \mathbf{Z_{12}}$. *Which elements are in the kernel* $\mathrm{Ker}\phi$ *of* ϕ *?*

Define a homomorphism $\psi : \mathbf{Z_{12}} \to S(4)$ *whose kernel is the subgroup of order* 6 *of* $\mathbf{Z_{12}}$, *and a homomorphism* $\eta : \mathbf{Z_{12}} \to S(4)$ *whose kernel is the subgroup of order* 4. *Is there a homomorphism* $\theta : \mathbf{Z_{12}} \to S(4)$ *whose kernel is the subgroup of order* 2 *? Justify.*

Exercise 5. *Describe the lattice of all subgroups of*
(i) $\mathbf{Z_6}$;
(ii) $\mathbf{Z_8}$;
(iii) $\mathbf{Z_{12}}$;
(iv) $S(3) = \langle a, b \mid a^3 = b^2 = (ab)^2 = 1 \rangle$;
(v) $D(8) = \langle a, b \mid a^4 = b^2 = (ab)^2 = 1 \rangle$.

Exercise 6. *Give necessary and sufficient conditions for* m *and* n *so that there exists a homomorphism* $\phi : \mathbf{Z_m} \to \mathbf{Z_n}$ *that is*
(i) nontrivial (i.e. $\phi(\mathbf{Z_m}) \neq 0$*); (ii) injective; (iii) surjective.*

Exercise ♠ *List all* 24 *elements of* $S(4)$ *as products of the transpositions* $(12), (23)$ *and* (34). *Show that each element is a product of at most* 6 *factors. Identify the subgroup* $A(4)$ *of all even permutations. Show that* $A(4)$ *has* 8 *proper (non-trivial) subgroups.*

Solutions will be sent to all students by e-mail.
They will be also available in the display case opposite of my office 4205HP
on Monday, March 21, 2005.