HOMEWORK No 2 (January 19, 2005)

Exercise 1. *A notice board, 86 cm wide, is to be divided into two types of columns, 34 mm and 38 mm wide, respectively. Is a division, without any space being wasted, possible ? Justify your answer.*

Exercise 2. *Give a necessary and sufficient condition for the quadratic equation*

$$x^2 + x + [a] = [0]$$

to have a solution in the field Z_{13}.

Exercise 3. a) *If* 17 *divides* n^{100}, *where* $n \geq 2$ *is natural, show that the quotient* $\frac{n^{100}}{17}$ *is divisible by the number* 17^{99}.

b) *Let* n *be a natural number and* $N = (n+5)^3 - n^3$. *Show that* 5 *is a divisor of* N. *Moreover, show that if* 5^2 *divides* N, *then also* 5^3 *divides* N.

c) *If* p *is prime and* $p \,|\, n^k$, *is it true that* $p^k \,|\, n^k$ *? Justify your answer.*

Exercise 4. *Let* a *be a natural number and* p *a prime that does not divide* a.

a) *Show that the set* S *of the natural numbers* k *such that* $a^k \equiv 1 \,(mod p)$ *is non-empty.*

b) *Denote the least element of* S *by* s.*If* k *satisfies* $a^k \equiv 1 \,(mod p)$ *and* $k = sq + r, 0 \leq r < s$, *show that* $a^r \equiv 1 \,(mod p)$. *Deduce that* $r = 0$, *and thus* $s \,|\, k$.

c) *Prove that* s *divides* $p - 1$.

[s is the **order** of the element a modulo p.]

♠**Exercise.** *How many zeroes are at the end of* $1000\,!$ *(thousand factorial, in the decimal system)?*

If you can't solve an exercise, you can always look up the answer.

But try first to solve it by yourself - this way you'll learn more and you'll learn faster.

Solutions will be available in the display case opposite of my office 4205HP

on Monday, January 24, 2005.