# HOMEWORK No 1 (January 12, 2005)

Exercise 1. (a) *Show that for any natural* $n$*,*

$$2 \times 6 \times 10 \times ... \times (4n-2) = (n+1)(n+2)...2n.$$

(b) *Prove that a set* $X$ *with* $n$ *elements has exactly* $2^n$ *subsets.*

Exercise 2. *Let* $a$ *and* $b$ *be natural numbers. Let* $D$ *and* $M$ *be their greatest common divisor and the least common multiple, respectively. Prove that*

$$DM = ab.$$

Exercise 3. *Find the greatest common divisor* $D = (76729, 111077)$ *and express* $D$ *as a linear combination of 76729 and 111077.*

Exercise 4. *Draw the lattice of the positive divisors of 3,600.*

Exercise 5. *Prove that for any natural* $n$ *and* $k$ *such that* $k \leq n$*,*

$$\binom{n}{k} = \sum_{t=1}^{n-k+1} \binom{n-t}{k-1}.$$

♠ Exercise. *For any natural* $n$ *and* $k$ *such that* $k \leq n$*, solve the equation*

$$x^2 - \binom{n}{k}x + \binom{n-1}{k-1}\binom{n-1}{k} = 0.$$

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**If you can't solve an exercise, you can always look up the answer.**

**But try first to solve it by yourself - this way you'll learn more and you'll learn faster.**

Solutions will be available in the display case opposite of my office 4205HP

**on Monday, January 17, 2005.**