Computational (or short-cut formula) for the variance, covariance, and correlation

$$\sigma^2 = \mathbf{E}(X^2) - [\mathbf{E}(X)]^2, \qquad \mathrm{Cov}(X,Y) = \mathbf{E}(XY) - \mathbf{E}(X)\mathbf{E}(Y), \qquad \rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}.$$

TABLE OF COMMON DISTRIBUTIONS

Binomial(n, p)	
pmf	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$
mean and variance	$\mathbf{E}(X) = np, \quad \mathbf{Var}(X) = np(1-p)$
Hypergeometric(n,M,N)	
pmf	$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, \quad \max(0, n-N+M) \le x \le \min(n, M)$
mean and variance	$\mathbf{E}(X) = n\frac{M}{N}, \quad \mathbf{Var}(X) = \left(\frac{N-n}{N-1}\right) n \frac{M}{N}\left(1 - \frac{M}{N}\right)$
Negative binomial(r,p)	
pmf	$p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots$
mean and variance	$\mathbf{E}(X) = \frac{r(1-p)}{p}, \quad \mathbf{Var}(X) = \frac{r(1-p)}{p^2}$
Poisson($\boldsymbol{\lambda}$)	
pmf	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$
mean and variance	$\mathbf{E}(X) = \lambda, \quad \mathbf{Var}(X) = \lambda$
Discrete uniform	
pmf	$p(x) = 1/k, \quad x = 1, 2, \ldots, k$
mean and variance	$\mathbf{E}(X) = \frac{k+1}{2}, \quad \mathbf{Var}(X) = \frac{(k+1)(k-1)}{12}$

Normal($\boldsymbol{\mu}, \boldsymbol{\sigma^2}$)	
pdf	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad x \in \mathbb{R}$
mean and variance	$\mathbf{E}(X) = \mu, \quad \mathbf{Var}(X) = \sigma^2$
Gamma($\boldsymbol{\alpha}, \boldsymbol{\beta}$)	
pdf	$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \quad x \ge 0$
mean and variance	$\mathbf{E}(X) = \alpha\beta, \quad \mathbf{Var}(X) = \alpha\beta^2$
Exponential($\boldsymbol{\lambda}$)	
pdf	$f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$
mean and variance	$\mathbf{E}(X) = 1/\lambda, \quad \mathbf{Var}(X) = 1/\lambda^2$
Chi Squared(n)	
pdf	$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{(n/2)-1} e^{-x/2}, \quad x \ge 0$
mean and variance	$\mathbf{E}(X) = n, \quad \mathbf{Var}(X) = 2n$
Uniform(a, b)	
pdf	$f(x) = \frac{1}{b-a}, \quad a \le x \le b$
mean and variance	$\mathbf{E}(X) = (a+b)/2, \quad \mathbf{Var}(X) = (b-a)^2/12$

Rule of thumb If sample size $n > 30$, then the Central Limit Theorem (CLT) can be applied.

DESCRIPTIVE STATISTICS

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} X_i^2 - \frac{(\sum_{i=1}^{n} X_i)^2}{n}\right]$$

TEST STATISTICS

$$Z = \sqrt{n}\,\frac{\overline{X} - \mu_0}{\sigma}, \qquad T = \sqrt{n}\,\frac{\overline{X} - \mu_0}{S}, \qquad Z = \sqrt{n}\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)}}$$

CONFIDENCE INTERVALS

CI's for μ (σ is known)

$$\left(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$\left(-\infty, \overline{X} + z_{\alpha}\frac{\sigma}{\sqrt{n}}\right) \quad \text{and} \quad \left(\overline{X} - z_{\alpha}\frac{\sigma}{\sqrt{n}}, +\infty\right)$$

CI's for μ (σ is unknown)

$$\left(\overline{X} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}, \overline{X} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}\right)$$

$$\left(-\infty, \overline{X} + t_{\alpha,n-1}\frac{S}{\sqrt{n}}\right) \quad \text{and} \quad \left(\overline{X} - t_{\alpha,n-1}\frac{S}{\sqrt{n}}, +\infty\right)$$

CI for σ^2

$$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}\right)$$

$$\left(0, \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}\right) \quad \text{and} \quad \left(\frac{(n-1)S^2}{\chi^2_{\alpha,n-1}}, +\infty\right)$$

Large-sample CI for p

$$\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

Large-sample CI for arbitrary θ

$$\left(\hat{\theta} - z_{\alpha/2}S_{\hat{\theta}}, \hat{\theta} + z_{\alpha/2}S_{\hat{\theta}}\right)$$

where $S_{\hat{\theta}}$ is an estimator of the standard deviation $\sigma_{\hat{\theta}}$ of $\hat{\theta}$