My research focuses on the study of deterministic and stochastic differential equations and their applications in electromagnetism, fluid dynamics in particular geophysical fluid dynamics and hydrodynamic stability, and the dynamics of the ionosphere. I use different mathematical tools from real and complex analysis, stochastic analysis, asymptotic and perturbation methods such as WKB method, weakly nonlinear theory, Fourier transform and Laplace transform methods, numerical methods that include but not limited to predictor-corrector methods, higher order numerical schemes, Wiener chaos expansion and pseudo-spectral methods [2, 4, 5, 9, 11, 12, 13].

I also use special functions and their asymptotic expansions to evaluate non-elementary (definite and indefinite) integrals [6, 8] and to study some problems in probability, statistics and science [1, 6, 7, 8, 9].

1 Dynamics of the ionosphere and the neutral atmosphere

I study the electrodynamics of the ionosphere (see next section). Based on the fluid dynamics' laws of conservation of mass, momentum and energy, I model ionospheric disturbances and their interactions with the lower atmosphere in terms of deterministic partial differential equations (PDEs) and stochastic partial differential equations (SPDEs). The electrodynamic processes (the Lorentz force and the Joule heating) are taken into account in the fluid dynamics equations. My ultimate goal is to compare analytical and numerical simulation results with observations, see for example [2, 5].

Some of my related future work

- Numerical modelling of ionospheric disturbances propagating downward that reach the neutral atmosphere and then interact with it and generate travelling atmospheric disturbances (TADs).
- Investigate the effects of critical layers on ionospheric disturbances.
- Investigate other wavy phenomena such as tidal disturbances in the ionosphere.

2 Electromagnetics in time-dependent media

When electromagnetic fields interact with a time-dependent medium, the properties of the medium (e.g. permittivity, permeability, conductivity) evolve with time, and the medium becomes dispersive. I am interested in the problem of electromagnetic field interactions in time-dependent media, particularly media that fluctuate randomly in time. In this case, the properties of the medium (permittivity, permeability, conductivity) are random functions of time, and the interactions are modeled by (stochastic) wave equations with stochastic coefficients.

For example, due to magnetic storms, the upper atmosphere (ionosphere) becomes a random medium because of the presence of the random scatters. Taking the ionosphere as a weakly time-dependent random medium, in [5], we have obtained a formula to calculate the electromotive force (EMF) induced by the magnetic storm in terms of the ionospheric disturbance characteristics (wavenumber or wavelength) by means of Faraday's law of induction.

I am also interested in the study of electromagnetic field interactions in dispersive media. In [4], I model such interactions in a time-dependent linear isotropic medium, and obtain analytical solutions for some particular cases of the properties of the time-dependent medium. For example, the permittivity and the permeability are exponential functions of time and sinusoidal functions of time.

Some of my related future work

- Take into consideration the nonlinear effects in the interactions.
- Investigate electromagnetic wave interactions in dispersive conducting time-dependent media.
- Investigate related problems in anisotropic media.

3 Analytical and numerical methods on finite domains

In computational fluid dynamics CFD, for example, it is important to apply non-reflecting boundary conditions to prevent reflective effects, e.g. wave reflections, or to avoid instabilities at the outflow (exit) boundary. I have numerically implemented the linear time-dependent boundary condition developed by [16] for modeling internal gravity waves. I found that, in the presence of nonlinearities, instabilities keep developing at the outflow boundary. My ultimate goal consisted of expanding the linear radiation condition in [16] to include a nonlinear correction to represent nonlinear interactions at the outflow boundary, see [12].

To assess the accuracy of numerical methods in geophysical fluid dynamics (GFD), it is important to compare numerical solutions with exact solutions usually obtained for simple configurations where nonlinear and viscous effects are neglected. To that end, I derived exact expressions for long Rossby waves and long gravity waves that can be used to assess the accuracy of numerical method in GFD [13, 14]. For instance, I assessed the accuracy of the numerical solutions in [12] by comparing these to the analytical solutions derived in [14].

As a graduate student, I have examined evolution SPDEs driven by Brownian motion (eg. stochastic heat equation, stochastic convection-diffusion equation and stochastic Burgers equations) and methods for their numerical solutions, and used numerical methods based on the Wiener chaos expansion (WCE) (see my doctoral thesis [15]). Recently, I have applied a WCE based numerical method to approximate the solutions of the stochastic generalized Kuramoto-Sivashinsky (SgKS) equation driven by Brownian motion forcing [6]. Currently, I am interesting in the numerical solutions of initial boundary value problems (IBVPs) involving SPDEs which are subject to nonhomogeneous stochastic boundary conditions and stochastic initial conditions.

Some of my related future work

- Develop and numerically implement time-dependent non-reflecting boundary conditions for complex atmospheric perturbations such as Kelvin waves.
- Derive boundary conditions for the numerical solutions of travelling ionospheric disturbances (TIDs) and travelling atmospheric disturbances (TADs) on bounded domains.
- Develop numerical methods or use or improve existing numerical methods for solving stochastic evolution equations.

4 Hydrodynamic stability

Stability and transition in shear flows are mechanisms which need to be understood importantly for applications in mechanical and aerospace engineering and in the atmospheric science. I am working on the stability of viscous flows in boundary layers by examining the solutions of the Orr-Sommerfeld in two and three dimensions. I use analytical and asymptotic methods (e.g., weakly nonlinear methods and WKB methods) [3].

Some of my related future work

- Take into account time dependence in the Orr-Summerfeld equation.
- Solve the Orr-Sommerfeld equation with a forcing term, and in particular, a random forcing for turbulence modelling.

5 Evaluation of non-elementary integrals, special functions and applications

An example of a non-elementary integral is $\int e^{-x^2}dx$. This integral cannot neither be expressed in terms of elementary functions such polynomials of finite degree, exponentials and logarithms nor their algebraic combinations. And to my knowledge, there did not exist a formula for the antiderivative of e^{-x^2}. To compute the area under the Gaussian, $\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi}$, one has to write the integral as a double integral and then use polar coordinates. In [7, 8], I write the integrand in terms of its Taylor series, integrate the series term by term, write the resulting series in terms of a special function and then use the asymptotic expansion of the special function. For instance, I show that

$$\int_{-\infty}^{+\infty} e^{-\beta^2 x^\alpha}dx = \frac{2}{\beta^{\frac{2}{\alpha}}}\Gamma\left(\frac{1}{\alpha}+1\right), \alpha \geq 2 \text{ and even}, \beta > 0, \tag{1}$$

and

$$\int\limits_{-\infty}^{+\infty} \frac{\sin(\lambda x^\alpha)}{\lambda x^\alpha}dx = \frac{\Gamma\left(\frac{1}{2\alpha}+1\right)}{\Gamma\left(\frac{3}{2}-\frac{1}{2\alpha}\right)}\frac{\sqrt{\pi}}{2}, \alpha \geq 1. \tag{2}$$

I also use special functions and their asymptotic approximations to examine the statistics of the coincidence event and use it to construct confidence intervals [8, 10, 11]. I also use special functions and their asymptotic approximations to solve some science related problems [1, 8].

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