Stochastic, Mean-Field and Network Systems, and Beyond

Workshop in honour of the 80th birthday of Professor Peter E. Caines

May 10, 2025

Room A.436 Fonds Cogeco, HEC Montreal, Hélène-Desmarais Building, 501 Rue De la Gauchetiere Ouest, QC, Canada H2Z 1Z5

Workshop Schedule

Morning Session (8:20 AM – 12:00 PM)

Time	Speaker	Duration	Notes
7:40–8:20 AM		40 mins	*Breakfast*
8:20–8:30 AM		10 mins	*Opening Remarks*
8:30–9:00 AM	Aditya Mahajan	30 mins	TBD
9:00–9:30 AM	Malcolm C. Smith	30 mins	The Case Against Process Noise: Input and State Estimation Without Priors
9:30–10:00 AM	Bozenna Pasik-Duncan & Tyrone E. Duncan	30 mins	TBD
10:00–10:30 AM		30 mins	*Coffee Break*
10:30–11:00 AM	Serdar Yüksel	30 mins	Kernel Topologies in Stochastic Control and Implications for Approximations, Robustness, and Learning
11:00–11:30 AM	Roland P. Malhamé	30 mins	On the Role of Observations and Communications in Propagating Mean Field Controls
11:30–noon AM	Karen Rudie	30 mins	Enforcing Opacity to Maintain Secrecy and Security in Discrete-Event Systems

Lunch Break (12:00 PM – 1:30 PM)

Afternoon Session (1:30 PM – 5:00 PM)

Time	Speaker	Duration	Notes
1:30–2:00 PM	Jun Liu	30 mins	From COCOLOG to Neural Certificates: A Journey Toward Formal Correctness in Control
2:00–2:30 PM	Prashant G. Mehta	30 mins	Mathematics for Transformer Modeling
2:30–3:00 PM	Tryphon T. Georgiou	30 mins	Controllability via State Feedback, Redux *(Online)*
3:00–3:30 PM		30 mins	*Coffee Break*
3:30–3:50 PM	Minyi Huang	20 mins	Mean Field Games on Networks: from Dense to Sparse (Recent work with Prof. Peter E. Caines)
3:50–4:10 PM	Ali Pakniyat	20 mins	Unnormalized Distributions: Measure-Theoretic Optimal Control of Hybrid Mean Field Systems
4:10–4:40 PM	Dena Firoozi	30 mins	Quantilized Mean Field Games: Early-Stage Venture Investments
4:40–5:10 PM	Steven W. Zucker	30 mins	Machine Learning, Neuroscience, and Games

Detailed Information of the Talks

- **Title**: Enforcing Opacity to Maintain Secrecy and Security in Discrete-Event Systems

  Speaker: Karen Rudie, Queen's University

  Abstract: The control theory of discrete-event systems (DESs) is a modeling framework for capturing the ordering of events or actions. Discrete-event systems modeling can be complementary to traditional continuous-time systems modeling or can be used alongside or in concert with continuous-time modeling in hybrid systems. Since decision-making is tantamount to prescribing which actions should or should not happen or which actions should happen before others, the body of work in DES theory is well-positioned to allow us to tackle security problems in cyber-physical systems. In this talk we focus on the notion of opacity, which is the property of ensuring that secret states or secret sequences of events are not discernible from non-secret states or events to a hostile agent. Using a motivating example from an industry project on autonomous vehicles searching unknown terrain, we show how control can be used to enforce opacity.

- **Title**: Machine Learning, Neuroscience, and Games

  Speaker: Steven W. Zucker, Yale University

  Abstract: We organize the responses of neurons in the mouse's visual system using manifold inference, showing which neurons act similarly (in time) to different components of a stimulus ensemble. The topology of the resulting manifolds suggests neurons can act differently depending on local context, likely violating mean-field assumptions.

- **Title**: From COCOLOG to Neural Certificates: A Journey Toward Formal Correctness in Control

  Speaker: Jun Liu, University of Waterloo

  Abstract: This talk traces a path from early logic-based control frameworks – most notably COCOLOG and its hierarchical extension – to modern abstraction and synthesis techniques for nonlinear systems, as well as to recent developments in verifiable learning with physics-informed neural networks for stability and control. We highlight the pioneering work of Professor Peter Caines on dynamically consistent abstractions and show how these ideas anticipated later developments in hybrid systems and formal methods. Building on these foundations, we present recent advances in robustness-aware control synthesis and neural Lyapunov certificates, and point to the emerging potential for unifying logic, learning, and verification in the pursuit of formally correct control.

- **Title**: On the Role of Observations and Communications in Propagating Mean Field Controls

  Speaker: Roland Malhamé, GERAD and Ecole Polytechnique Montréal

  Abstract: What is by now "classical" mean field game theory has started with an assumption of large-scale games of agents weakly coupled via a network implicitly considered at the outset as completely connected. With this assumption, even in its more recent network generalizations, the theory was developed while ignoring the possible time scale separation between an agent dynamics and the speed at which she can collect information about other agents whose state can influence her control decisions. We revisit the classical linear quadratic mean field game problem under the conditions of agents being able to communicate information (states or otherwise) only via a fast operating communication network which is connected, but not completely connected. Agents are assumed to achieve empirical mean state estimation via a consensus-based algorithm which produces reliable results, albeit only after a fixed time delay. Agents build their control in a piecewise open loop fashion. They update their control policy only after obtaining sampled delayed empirical mean state estimates. The resulting hybrid dynamic programming problem solution is presented and its performance is numerically evaluated.

  This is joint work with Farid Rajabali and Shuang Gao

- **Title**: The Case Against Process Noise: Input and State Estimation Without Priors

  Speaker: Malcolm C. Smith, University of Cambridge

  Abstract: The talk is based on recent work with G. Gakis. The classical Kalman filter problem formulation imposes symmetrical assumptions on the process and measurement noise. This work considers the case where the assumption that the exogenous inputs are Gaussian with known mean and covariance is unrealistic. In such cases a model-based approach is required which places the unknown inputs and states on an equal footing in filtering and smoothing problems. The solution of this problem will be examined with various approaches: a zero informational limit of a regular Kalman filter; a direct least-squares solution; and a stochastic formulation.

- **Title** : Kernel Topologies in Stochastic Control and Implications for Approximations, Robustness, and Learning

  Speaker: Serdar Yüksel, Queen's University

  Abstract : Stochastic kernels represent system models, control policies, and measurement channels, and thus offer a general mathematical framework for learning, robustness, and approximation analysis. To this end, we will first present and study several kernel topologies. These include weak* (also called Borkar) topology, Young topology, kernel mean embedding topologies, and strong convergence topologies. Convergence, continuity, and robustness properties of optimal cost on models and policies will be presented in both discrete-time and continuous-time stochastic control:

  On models viewed as kernels; we study robustness to model perturbations, including finite approximations for discrete-time models and robustness to more general modeling errors and study the mismatch loss of optimal control policies designed for incorrect models applied to a true system, as the incorrect model approaches the true model under a variety of kernel convergence criteria: We, in particular, show that the expected induced cost is robust under continuous weak convergence of transition kernels. Under stronger Wasserstein or total variation regularity, a modulus of continuity is also applicable. As applications of robustness under continuous weak convergence via data-driven model learning, (i) robustness to empirical model learning for discounted and average cost criteria is obtained with sample complexity bounds; and (ii) convergence and near optimality of a quantized Q-learning algorithm for MDPs with standard Borel spaces, which we show to be converging to an optimal solution of an approximate model under both discounted and average cost criteria, is established.

  In the context of continuous-time models, we obtain counterparts where we show continuity of cost in policy under Young and Borkar topologies, and robustness of optimal cost in models including discrete-time approximations for finite horizon and infinite-horizon discounted/average cost criteria. Discrete-time approximations under several criteria and information structures will then be obtained via a unified approach of policy and model convergence.

  A conclusion is that weak topologies are appropriate for policies and strong topologies are so for models (both viewed as kernels) towards establishing very general existence, approximations, robustness and learning results.

  Joint work with Ali D. Kara, Somnath Pradhan, Naci Saldi, Omar Mrani-Zentar, and Tamas Linder.

- **Title**: Controllability via State Feedback, Redux

  Speaker: Tryphon T. Georgiou, University of California, Irvine

  Abstract. The purpose of the talk is to revisit the question of controllability of the state-transition matrix for linear dynamics effected via time-varying state feedback gain K_t. That is, we consider the feedback gain as constituting our sole control authority for the dynamics $\dot{\Phi}_t = (A - BK_t)\Phi_t$. In spite of the fact that the system is no longer linear, due to the bilinear entry $K_t\Phi_t$, we show that it is strongly controllable if and only if the Kalman rank condition holds for the pair (A,B). To this end we correct and extend an earlier attempt by Roger Brockett to establish the same. The subject is pertinent in steering a collection of particles that obey identical dynamics by broadcasting a control protocol in K_t that must be simultaneously implemented by all. We further note that a related conjecture by Roger Brockett, that there exists a protocol that is continuous in the problem data, fails due to an inherent topological obstraction in steering state transition matrices between terminal specifications.

  Joint work with Mahmoud Abdelgalil, University of California, San Diego.

- **Title**: Unnormalized Distributions: Measure-Theoretic Optimal Control of Hybrid Mean Field Systems

  Speaker: Ali Pakniyat, University of Alabama

  Abstract: Classical mean field control theory relies on normalized probability distributions to describe large-scale multi-agent systems, but this approach breaks down for hybrid systems where agents switch between discrete modes and, hence, population sizes within each mode vary dynamically. In this work, I present a measure-theoretic framework for stochastic hybrid mean field systems, where unnormalized occupation measures serve as the fundamental objects for capturing both the evolving population counts and the complex inter-modal and intra-modal interactions driven by controlled and autonomous switchings. By embedding the optimal control problem within a convex optimization problem over measures and leveraging convex duality relations between the space of measures and that of continuous functions, we address the limitations of probability-based mean field models and provide a rigorous methodology for hybrid mean field systems.

- **Title**: Mathematics for Transformer Modeling

  Speaker: Prashant G. Mehta, University of Illinois Urbana-Champaign

  Abstract: I will describe a mathematical framework for causal nonlinear prediction in settings where observations are generated from an underlying hidden Markov model (HMM). Both the problem formulation and the proposed solution are motivated by the decoder-only transformer architecture, in which a finite sequence of observations (tokens) is mapped to the conditional probability of the next token. I will describe these correspondences as part of my talk.

- **Title**: Quantilized Mean Field Games: Early-Stage Venture Investments

  Speaker: Dena Firoozi, HEC Montreal

  Abstract: Mean field game (MFG) theory aims to understand and analyze the behavior of complex systems, where many competitive individuals interact, and their collective behavior significantly impacts the overall system dynamics. In such systems, each agent's decision (control) depends not only on its own situation (state) but also on the collective behavior (distribution) of all agents. MFG models typically involve the distribution of state or control across the agent population, which reduces to the inclusion of the mean value of the distribution in linear-quadratic models. In this work, we present a MFG model where the quantiles of the agents' state distribution are incorporated into the model following recent works by Caines, Malhamé, et al. We address such MFG problems and discuss their implications within the context of early-stage venture investments.

- **Title**: TBD

  Speaker: Bozenna Pasik-Duncan, University of Kansas

  Abstract: TBD

- **Title**: TBD

  Speaker: Tyrone E. Duncan, University of Kansas

  Abstract: TBD

- **Title**: Mean Field Games on Networks: from Dense to Sparse (Recent work with Prof. Peter E. Caines)

  Speaker: Minyi Huang, Carleton University

  Abstract: This talk shares some wonderful experiences when working with Prof Caines in recent years, often together in Ottawa. When mean field game (MFG) theory and graphon theory crossed paths, Prof Caines advocated a new theory – mean field games on dense networks. The next question is what can be done on sparse networks, with background from natural resource extraction (such as groundwater pumping) and markets of retail goods. We aim to obtain a meaningful mean field model allowing to capture influence of neighboring subpopulations in the large sparse network limit. We will explain how to derive second order interactions via Laplexion dynamics. Subsequently, the HJB-FPK formalism in standard MFG theory can be extended to the sparse network case (Caines and Huang, CDC'25 submitted).

Location Information:

Workshop & coffee breaks: A.436 Salle de cours Fonds Cogeco, 501 Rue De la Gauchetiere Ouest
Lunch: A.878 Salle de réception Guy-Fréchette, 501 Rue De la Gauchetiere Ouest
Dinner Location: 2nd Floor, La Maison Grecque, 450 Duluth Ave E, Montreal, Quebec H2L 1A5
Dinner Start Time: 6:30pm

Organizing Committee:

Dena Firoozi (HEC Montreal), Shuang Gao (Polytechnique Montreal), Minyi Huang (Carleton), and Aditya Mahajan (McGill)

Website:

https://people.math.carleton.ca/ mhuang/Workshop25/Stomns25a.html