The FIREBREAK Problem – *the backstory*

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Andrea C. Burgess
Jessica Enright
Jared Howell
David A. Pike
Brady Ryan

Some Questions for which answers aren’t usually obvious

- How does a paper happen?
- How does the research process unfold?
- How does the collaboration evolve?

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RESEARCH ARTICLE

WILEY

The firebreak problem

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1:MUN 2:UNB-SJ 3:Glasgow 4:MUN-Grenfell

Founded in 1999 as a Network of Centres of Excellence for the Mathematics of Information Technology and Complex Systems.

A key feature of its activities is to facilitate internships and partnerships between academic researchers and the private sector.

Date: Wed, 27 Feb 2013 11:34:46 -0330
From: Wolfgang Banzhaf <banzhaf@mun.ca>

Subject: Fwd: Potential MITACS Accelerate Internship Projects - seeking faculty supervisors and students

Faculty,
please have a look at these opportunities. If you have a interest in any one of these, please let Janice know asap!

Date: Tue, 26 Feb 2013 16:48:35 -0330

From: Hopkins, Janice <janiceh@mun.ca>

Hi Wolfgang,

Niraj Shukla, the new Director of Business Development, for MITACS, is seeking faculty interested in supervising graduate students on 3 potential MITACS Accelerate internship opportunities (minimum 4 months in duration). The internship for each project would be with a large local construction company.

Project 2 Remote Security Monitoring

Required Expertise: network architecture; and network security

Project Tasks:

1) Evaluate current solution for areas of improvement in video monitoring, and identify any bottlenecks/limitation that may impede scalability.

2) Identify and implement best network security practices

Date: Wed, 27 Feb 2013 11:44:43 -0330 (NST)

From: David Pike <dapike@mun.ca>

To: "Hopkins, Janice" <janiceh@mun.ca>

Hi Jan.

Project #2 sounds as though it might lend itself to being modelled as a graph in which a selection of vertices (a.k.a. "nodes") serve as monitoring locations. Depending on just what is wanted in an optimal solution, some variation of the graph theoretic problem of finding a dominating set of vertices might be applicable.

Anyway, I would be curious to hear more about the nature of the problem at hand.

- David.

So we met a few times and it sounded like what they wanted was something like this:

Imagine a big hardware store or a construction site. Theft is a concern, but monitoring devices can only be put in a few places. Where should they be put in order to best identify a thief? Incidents of theft may not be noticed in real time.

So we met a few times and it sounded like what they wanted was something like this:

Imagine a big hardware store or a construction site. Theft is a concern, but monitoring devices can only be put in a few places. Where should they be put in order to best identify a thief? Incidents of theft may not be noticed in real time.

My idea: model the landscape with a graph and put k cameras at whichever k vertices would create the most unmonitored regions. So when an item is noticed to be missing from an unmonitored area, we could then check the video records to see who passed through the nearby fields of vision.

Ultimately it turned out that what they wanted was not what I had in mind. But my misinterpretation seemed interesting in its own right.

Expressed as a formal decision problem:

KEY PLAYER PROBLEM

Instance: A graph G, an integer k, and an integer t.

Question: Does $V(G)$ contain a k-subset S such that $c(G - S) \geqslant t$?

$c(H)$ is the number of connected components of a graph H.

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This problem then got studied by MSc student Kathleen.

It is easily seen to be NP-complete. Interestingly though, when $k = \kappa(G)$ it can be solved in polynomial time.

Another situation:

A pathogen could emerge somewhere.

To prepare for a future outbreak, we can harden (in advance) a fixed selection of locations.

One approach would be to select a set of vertices as barriers between separated quarantine zones. Maybe we only have enough vaccine for k vertices, and we want to maximise the number of resulting quarantine areas.

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But what if, instead of preparing in advance for a future outbreak that could happen anywhere, we were in the position of having to respond after an outbreak has occurred at a specific location?

EURO 2015
12-15 JULY 2015
GLASGOW

27th European Conference on Operational Research

12-15 July 2015
University of Strathclyde

EURO The Association of European Operational Research Societies

THE OR SOCIETY

Optimal edge deletion for limiting the spread of contagion

Jessica Enright

> Real-world biological or information pathogens can be modelled as contagion spreading on contact graphs between agents. The control of contagion on graphs is therefore an area of both theoretical interest and critical practical importance. We describe our work on optimal link deletion in order to minimise potential outbreak size, with motivation and testing in an agricultural livestock disease setting.

Jess and I begin to chat.

August 2016: Jess visits Memorial University of Newfoundland

Jess gives a seminar presentation:

> Let vertex v be a heifer: two applications of graph theory to the epidemiology of Scottish cattle
>
> Graphs derived from contact data play an increasingly important role in disease control policy. Inspired by the graph derived from cattle trades in Scotland, I will talk about two applications of graph theory to controlling diseases of cattle (or other species, really, but my examples will be mainly bovine). The first is a method of finding a minimum edge deletion to limit maximum outbreak size, using a tree decomposition of limited treewidth. The second is a very simple method of calculating expected outbreak size on a dynamic graph using a breadth-first search ordering.

Jess, Andrea, Jared and I chat.

Through these visits and discussions we came to look at vertex deletion, with a goal of protecting as many other vertices as possible.

A temporal model already exists, in the form of the Firefighter Problem in which fire plays the role of the contagion, and a sustained response over time is permitted...

The Firefighter Problem (Bert Hartnell, 1995)

It is modelled as a game, played on a graph G.

There are two players: an arsonist and a brigade of k firefighters.

The arsonist goes first by selecting a vertex and setting it ablaze.

Now repeat:

- Each firefighter douses an unburnt vertex, forever protecting it.
- Fire then spreads to all unburnt unprotected vertices that are adjacent to a burning vertex.

The usual goal for the firefighters is to save as much of G as possible.

The goal for the arsonist is to select an initial vertex that results in maximum damage.

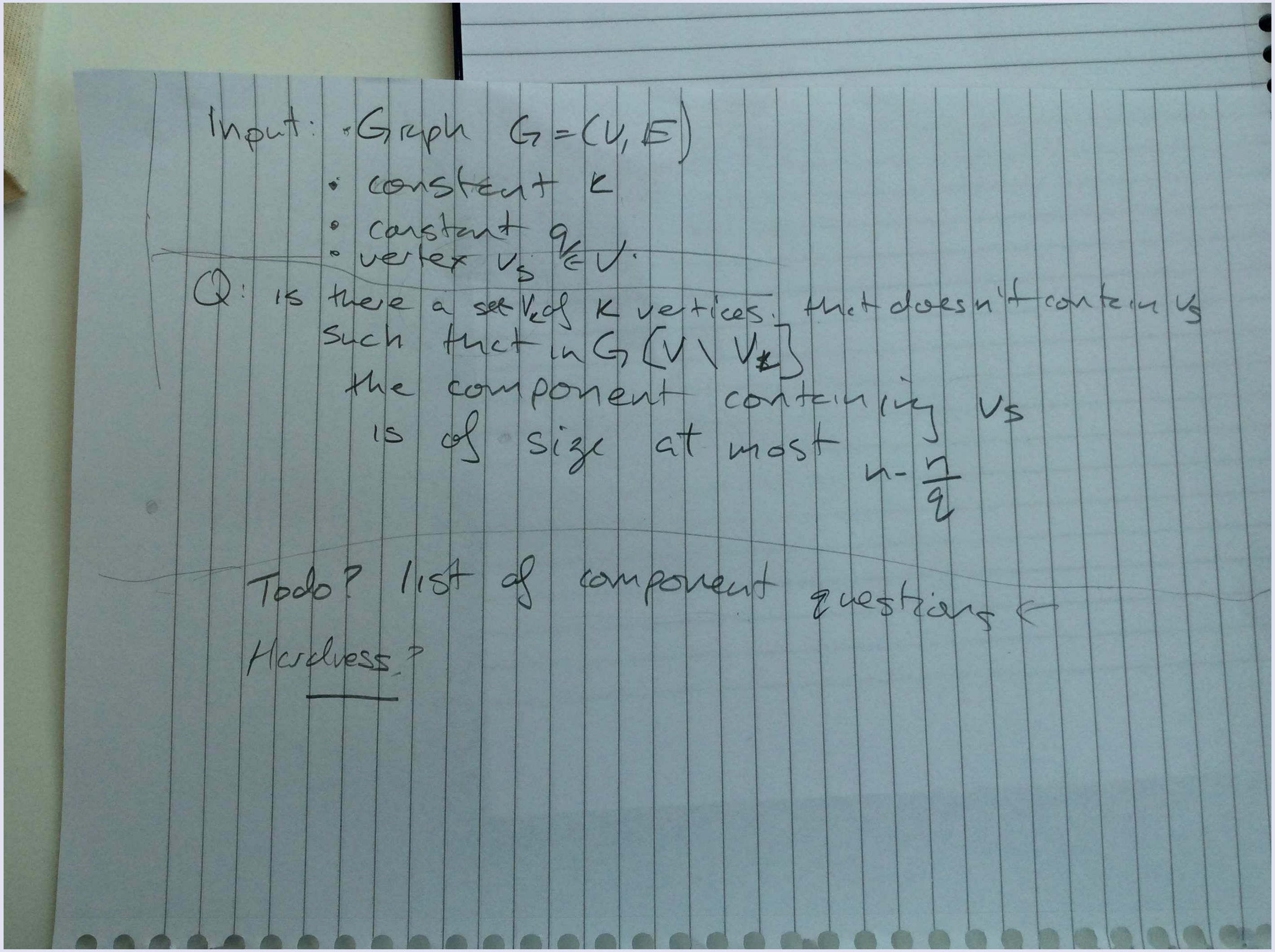
But what if an ongoing response is not an option?

What if the fire brigade can only act once?

That is, they form a firebreak in response to the fire,
and thereafter they take no further action.

July 2017 in Glasgow for the British Combinatorial Conference

Andrea, Jess and I chat some more. By this time we have formulated a version of the Firebreak Problem.



Our ultimate formulation:

FIREBREAK PROBLEM

Instance: A graph G, an integer k, an integer t,
and a vertex $v_f \in V(G)$.

Question: Does $V(G)$ contain a k-subset S such that $v_f \notin S$ and the number of vertices of $G - S$ that are separated from v_f is at least t ?

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How hard is this problem?

Is it NP-complete?

Are there classes of graphs for which it is in P?

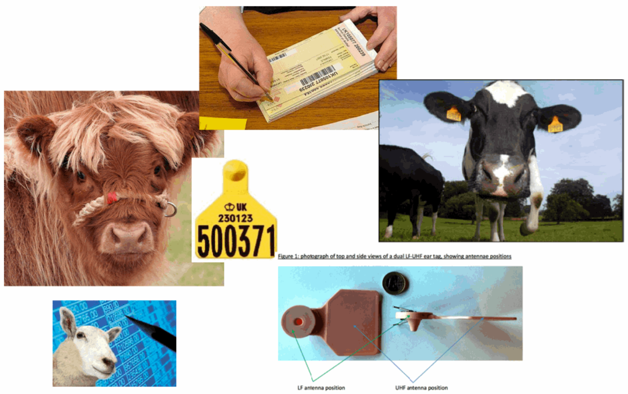
August 2017: GrasCAN in Corner Brook

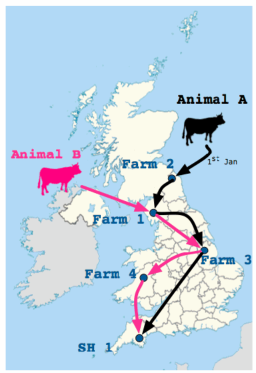
Jess gave a presentation about the Firebreak Problem.

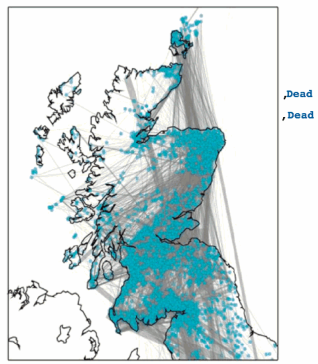
Why?

For **fun**.

But also: a bound on epidemiological interventions.





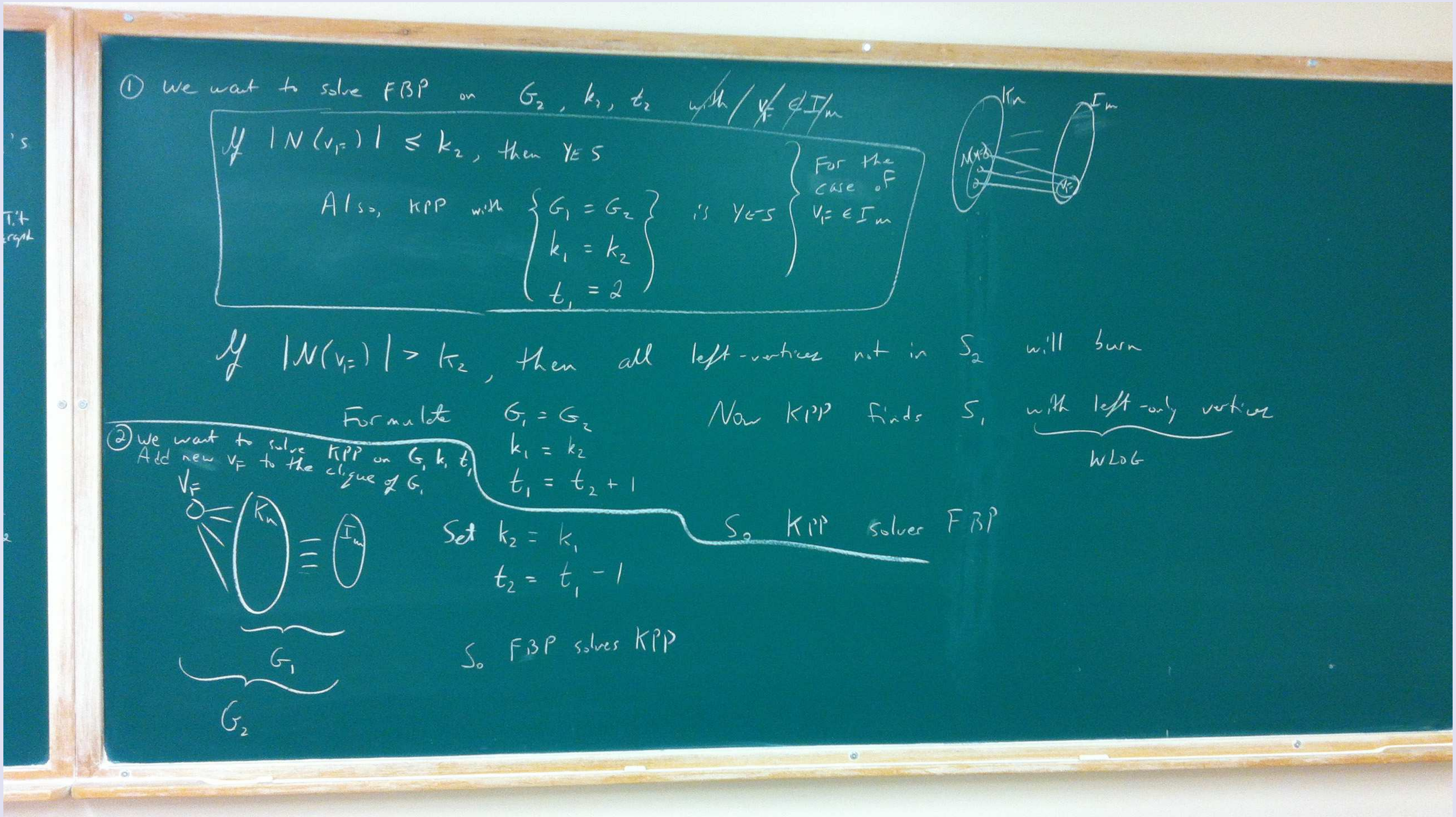


At this point we haven't yet realised that there's a connection with the Key Player Problem.

And we still don't know if the Firebreak Problem is NP-complete.

Andrea, Jared and I are in St. John’s a week later (August 2017)

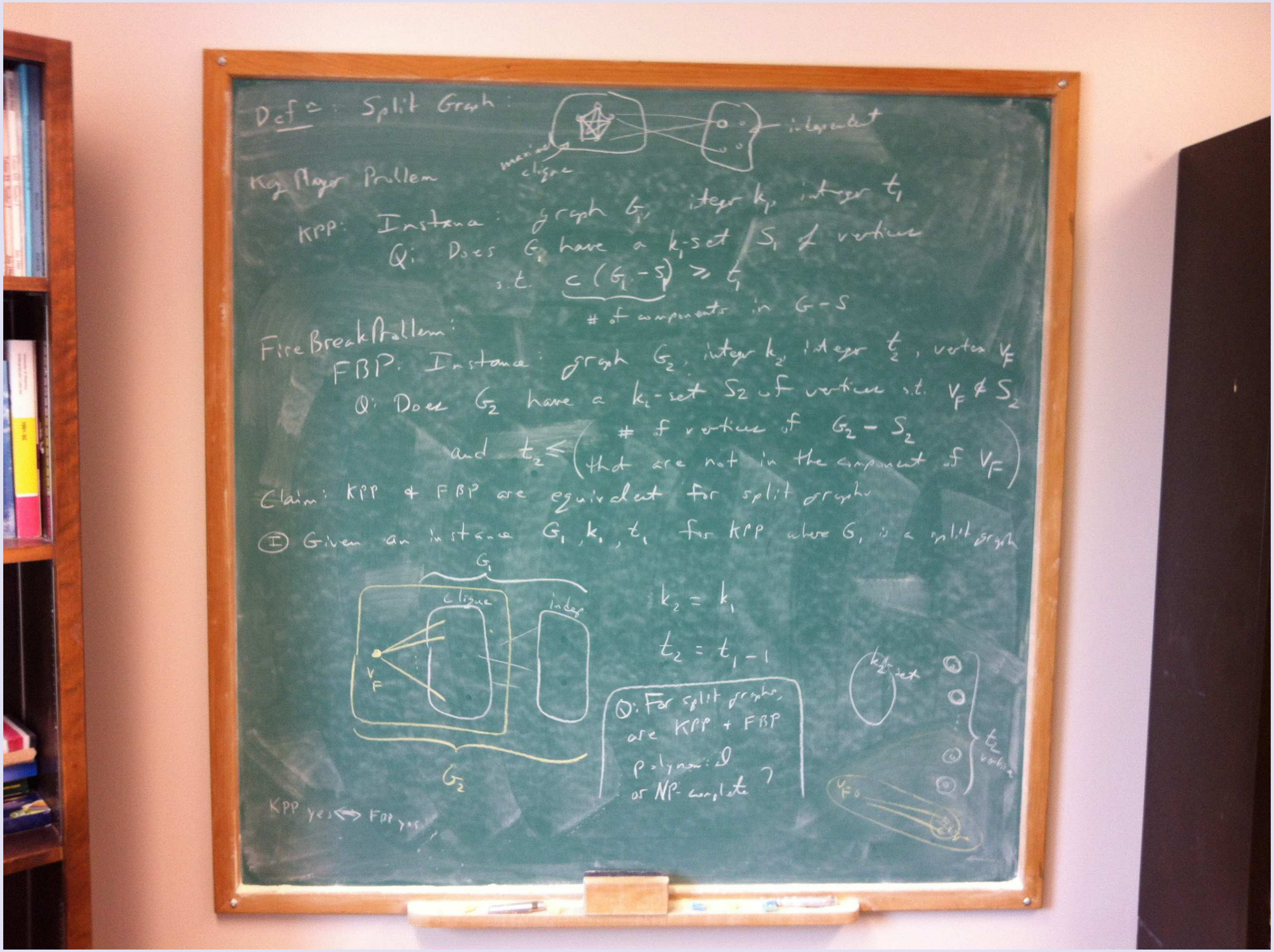
We show that the two problems are equivalent for split graphs.



But are the problems NP-complete on split graphs?

Summer 2018

Brady holds an NSERC USRA



Within days Brady established that the two problems are NP-complete on split graphs.

This relies on a paper published in 2014 in the journal *Networks*.

Among other things, Brady then looked at graphs having constant-bounded treewidth. We showed that the Firebreak problem is solvable in linear time on such graphs, but an actual algorithm eluded us.

Other graphs that we collectively looked at include bipartite graphs, cubic graphs, as well as intersection graphs.

The Manuscript

Collectively written and revised during the Fall of 2018

Submitted to *Networks* on 23 January 2019

Asked to revise on 26 January 2020

Revisions submitted on 11 May 2020

Accepted on 09 July 2020

Published online on 21 August 2020

Printed in the April 2021 issue (8 years after an initial idea)

Thank you.

Acknowledgements:

NSERC
CRSNG

Now over to Andrea...