Computer Science CSCC51 Spring 1999
Scarborough Campus University of Toronto

Homework Assignment #1
Due: Friday February 1 1999, in class
Worth: 8%

Please write your family and given names and **underline** your family name on the front page of your paper.

1. (20 points) A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in hours and the distance is in kms.

Time	0	1	4
Distance	0	70	352

(a) (5 points) Use Lagrange interpolation and the data in the above table to approximate the position of the car at times not listed in the table. Show the Lagrange basis functions and the interpolating polynomial in Lagrange form. Evaluate the polynomial at time = 3 hours. Transform the polynomial into monomial form (without solving the Vandermonde matrix).

(b) (5 points) Use Newton Divided Differences (NDDs) and the data in the above table to approximate the position of the car at times not listed in the table. Show the NDD table for the above data, the Newton basis functions and the interpolating polynomial in Newton form. Evaluate the polynomial at time = 3 hours. Transform the polynomial into monomial form (without solving the Vandermonde matrix). Compare with the polynomial in part (a) and explain.

(c) (3 points) Differentiate the polynomial in part (a) and use it as an approximation to the speed of the car. Use standard calculus techniques to find the maximum speed of the car in the interval $[0, 4]$. At what time is this maximum attained?

(d) (4 points) Add the point at time = 2, distance = 160 kms. Incorporate this extra piece of data in the table of part (b) doing **only** the necessary calculations. Show the calculations you have done and the (new) interpolating polynomial. Evaluate the polynomial at time = 3 hours doing **only** the necessary calculations.

(e) (3 points) Differentiate the polynomial in part (d) and use it as an approximation to the speed of the car. Use standard calculus techniques to find the maximum speed of the car in the interval $[0, 4]$. At what time is this maximum attained?

Note: This question is to be done by hand calculations.

2. (10 points) Assume now that we have the following values for the car of Question 1.

Time	0	1	4
Distance	0	70	352
Speed	0	82	88

Use Newton Divided Differences (NDDs) and the data in the above table to approximate the position of the car at times not listed in the table. Show the NDD table for the above data, the Newton basis functions and the Hermite interpolating polynomial in Newton form. Evaluate the polynomial at time = 3 hours. Transform the polynomial into monomial form (without solving the Vandermonde matrix).

Note: This question is to be done by hand calculations.

3. (25 points)

(a) (5 points) Write a MATLAB function `vanderh(x)` which returns the Vandermonde matrix arising from the **Hermite** interpolation relations, corresponding to points saved in the vector `x`. State in what order the coefficients of the respective interpolating polynomial are assumed to be.

(b) (20 points) Let `v` be a vector of 100 equidistant points in $[0, 4]$. (Use the built-in MATLAB function `linspace(a, b, n)` to create a set of n equidistant points in $[a, b]$.) Write a MATLAB script which

(b1) Uses the built-in MATLAB function `vander` to construct the Vandermonde matrix corresponding to the data in the table of Question 1, that is,

Time	0	1	4
Distance	0	70	352

Note that MATLAB builds the Vandermonde matrix with the columns in reverse order to that shown in class. Thus, the order of the coefficients of the respective interpolating polynomial is also reversed. Note also that MATLAB starts the index of vectors always from 1.

Constructs the right side of the interpolation relations accordingly.

Solves the resulting linear system to compute the coefficients of the interpolating polynomial in monomial form and outputs the coefficients.

Uses the built-in MATLAB function `polyval` to evaluate the polynomial on the points of `v` and saves the result in vector `p2`. Note that `polyval` assumes that the coefficients of the polynomial correspond to descending exponents of the monomials.

(b2) Repeats part (b1) of this question, for the data in the table of Question 1, including the data in part (d) of Question 1 and saves the result in vector `p3`.

(b3) Repeats part (b1) of this question, for the data in the table of Question 2, using your own function `vanderh(x)` and saves the result in vector `p5`.

(b4) Plots `p2, p3` and `p5` versus `v` in a single graph.

Note: To distinguish between three lines in the same plot use a dotted line for `p2`, a dashed one for `p3` and a solid for `p5`.

4. (25 points) Consider a polynomial of degree at most 3 that interpolates the data in the table of Question 1.

(a) (8 points) Write the polynomial in the form $p(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha t^3$. Write down the interpolating relations for the data in the table. Assuming that α is a free parameter, calculate α_0, α_1 and α_2. (Some of these may be given in terms of α.) How many polynomials of degree at most 3 interpolate the data in the table?

Note: This part is to be done by hand calculations.

(b) (4 points) Pick three values for α, namely, $\alpha = 1, -2$ and 0, and write down (by hand) the corresponding interpolating polynomials. Plot the three interpolating polynomials using 100 equidistant abscissae in the interval $[0, 4]$, in **one** plot.

Write down (by hand) the derivatives of the three interpolating polynomials. Plot the three derivatives using 100 equidistant abscissae in the interval $[0, 4]$, in **one** (another) plot.

Note: To distinguish between three lines in the same plot use a dotted line for $\alpha = 1$, a dashed one for $\alpha = -2$ and a solid for $\alpha = 0$.

(c) (13 points) Prove (mathematically) that the graphs of the three derivatives match at exactly two points in $[0, 4]$.

5. (20 points) Consider interpolating the function

$$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

by a polynomial $p(x)$ of degree at most n, using $n + 1$ distinct data points x_i, $i = 0, \ldots, n$, in $[-1, 1]$. Note that the data points may be anywhere in $[-1, 1]$.

(a) (12 points) In terms of n, give a (good) upper bound for

$$\max_{-1 \le x \le 1} \frac{|f(x) - p(x)|}{|f(x)|},$$

that holds for any choice of x_i, $i = 0, \ldots, n$. Explain how you arrived at the bound.

(b) (8 points) Give a numerical value to your bound of part (a) for $n = 4$, and $n = 5$.

General note: in all programming questions, hand in a hard-copy of your code, output and plots.