University of Toronto

CSCB70S—Data Structures and Techniques, Spring 1998

Assignment 4: Dynamic programming

Due: Wednesday April 8 at 10:00 am.
Worth: 10%.

Problem **1.** The *string cutting* problem is to break a string into several strings. The only allowable operation is to cut a string into two substrings. For a string of length n, a single cut takes n time units since it makes a copy of the string before cutting it. The running time does *not* depend upon the position of the cut.

Consider a string `abcdefghij`. Here are costs for two cuts which occur one after the other:

Strings	Action	Result	Cost
`abcdefghij`	cut `abcdefghij` after c	`abc defghij`	10
`abc defghij`	cut `defghij` after e	`abc de fghij`	7

A. Solve the string cutting problem in C using dynamic programming. Your program should receive as input a string S, its length n, and an array of m length for the fragments. You should check that such a cutting is possible; send an error message otherwise. The output must be the minimum cost cutting sequence and the fragments that produce such an optimal cut.

B. Suppose we want to cut the string `persuasiveness` into six fragments: $f_1, f_2, f_3, f_4, f_5, f_6$. The fragments have the following lengths and letters:

fragment	f_1	f_2	f_3	f_4	f_5	f_6
length	1	1	5	3	2	2
letters	`p`	`e`	`rsuas`	`ive`	`ne`	`ss`

You will use dynamic programming **by hand** to find the sequence of cuts that has minimal total cost. For that, you will fill in two tables with your intermediate results.

In the first table, the element in column c, row r contains the minimum total cost of cutting the subsequence $f_c, f_{c+1}, \ldots, f_{c+r}$. Note that row numbers increase upward.

In the second table, the element in column c, row r contains the index of the subsequence $f_c, f_{c+1}, \ldots, f_{c+r}$ after which the first cut should be done. For example, if the first cut of $f_3 f_4 f_5$ should be made after f_4, then the column 3, row 2 entry in the second table will contain a "4".

For the optimal sequence of cuts, give its cost and the cutting order.

What to hand in.

Submit your C program for part A. Test it with the example of part B and with another example of your choice. Use the submit command:

```
% submit -N a4 cscb70s *
```

Hand in copy of your source code, the output from the two tests and your manual solution of part B. For part B, show all your work, including all the values that you tested in determining each table element.

Problem **2.** Consider the problem of neatly printing a paragraph of text on a printer with fixed-width fonts. The input text is a sequence of n words containing $w_1, w_2, \ldots, w_n$ characters, respectively. Each line on the printer can hold up to M characters. If a printed line contains words i through j, then the number of blanks left at the end of the line (given that there is one blank between each pair of words) is

$$M - j + i - \sum_{k=i}^{j} w_k .$$

We want to minimize the sum, over all lines except the last, of the cubes of the number of blanks at the end of each line. For instance, if there are k lines with $b_1, b_2, \ldots, b_k$ blanks at the end, respectively, then we want to minimize $b_1^3 + b_2^3 + \cdots + b_{k-1}^3$.
Using dynamic programming, compute the arrangement of words on lines that minimizes the sum of the cubes. You have a paragraph with words whose lengths are $w_1, \ldots, w_7$:

word	w_1	w_2	w_3	w_4	w_5	w_6	w_7
length	5	7	2	3	5	4	9

Assume that each line can contain 12 characters.

COVER SHEET FOR ASSIGNMENT 4

Account name:

Surname:

Given Name(s):

Student Number:

Tutorial:

I declare that this assignment is my own work and is submitted in accordance with the University of Toronto Code of Behavior on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism described in the course syllabus.

I discussed this assignment with the following people:

Signature

GRADING

Problem 1		
Correctness	____________	/ 15
Output	____________	/ 5
Manual computations	____________	/ 10
Program style, including comments	____________	/ 5
Problem 2		
Correctness	____________	/ 10
Comments	____________	/ 5
Total	____________	/ 50