69.114 SAMPLE FINAL EXAM

YOU WILL SEE INSTRUCTIONS LIKE THIS: Fill in the appropriate answer ON THE SCANTRON SHEET with a **soft lead 2B pencil**. Be sure to fill in the parts of the SCANTRON SHEET with your name and student number too. This exam MUST be handed in also at the end of the examination, together with the SCANTRON sheet- but it is ONLY the SCANTRON sheet answers that will be marked. Only simple calculators are permitted.

1) If A and B are $n \times n$ matrices and $AB = BA$, then $(A + B)(A - B) = A^2 - B^2$
ANSWER: [A] True [B] False

2) If $A_{n\times n}$ is an invertible matrix, then the equation $Ax = b$ has a unique solution for all vectors $b_{n\times 1}$.
ANSWER: [A] True [B] False

3) Every finite product of $n \times n$ elementary matrices is an invertible matrix.
ANSWER: [A] True [B] False

4) Every straight line in the xy-plane is a subspace of $\Re^2$
ANSWER: [A] True [B] False

5) The matrix $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ is a product of 2 elementary matrices.
ANSWER: [A] True [B] False

6) Every set of $p + 1$ vectors $\{v_1, v_2, v_3, ..., v_{p+1}\}$ in R^p is linearly dependent.
ANSWER: [A] True [B] False

7) A is an $n \times n$ invertible matrix. Then, $|A| = -|A^{-1}|$.

ANSWER: [A] True [B] False

8) The columns of the 3×3 matrix $M = \begin{bmatrix} 5 & 5 & 10 \\ 10 & 0 & 10 \\ 0 & 10 & 10 \end{bmatrix}$ span $\mathcal{R}^3$
ANSWER: [A] True [B] False

9) A is a 4×5 matrix whose reduced row echelon form has exactly 3 pivot positions. Then $Ax = b$ has at least 1 solution for every choice of b where b is a 4×1 vector.
ANSWER: [A] True [B] False

10) Let T be a linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 12x_1 + 24x_2 \\ x_1 + 2x_2 \end{bmatrix}$. Then, T is an invertible linear transformation.

ANSWER: [A] **True** [B] **False**

11) A linear transformation $\mathcal{R}^n \to \mathcal{R}^m$ is completely determined by its effects on the columns of the identity matrix I_n.
ANSWER: [A] **True** [B] **False**

12) If $\lambda + 25$ is a factor of the characteristic polynomial of A, then 25 is an eigenvalue of A.
ANSWER: [A] **True** [B] **False**

13) If A is a diagonalizable matrix, and A has eigenvalues $\{0, -1\}$, then $A^2 = A$.
ANSWER: [A] **True** [B] **False**

14) Which of the following is the inverse matrix to the matrix $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$?

ANSWER:
[**A**] : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ [**B**] : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$ [**C**] : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

15) For $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $A_1 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 5 & 7 & 8 \\ 2 & 0 & 1 & -1 \\ 3 & 2 & 10 & 11 \end{bmatrix}$ the $(2,3)$ coefficient of the matrix product EA_1 is

ANSWER: [A] 17 [B] 0 [C] 7

The matrices U, W and vector b given here are referred to by questions 16-21, where W is the RREF of the augmented matrix $[U|b]$

$$U = \begin{bmatrix} 1 & -1 & 3 & 5 & 2 & 15 \\ 0 & 1 & -1 & -2 & 3 & 19 \\ -1 & -1 & -1 & -1 & 2 & 7 \\ 3 & 0 & 6 & 9 & 1 & 18 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 47 \\ 20 \\ -7 \\ 89 \end{bmatrix}, W = \left[\begin{array}{cccccc|c} 1 & 0 & 2 & 3 & 0 & 4 & 27 \\ 0 & 1 & -1 & -2 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 1 & 6 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

16) The solution set of $Ux = 0$ is a vector space of dimension
ANSWER: [A] 2 [B] 3 [C] 4

17) The linear transformation $T : \mathcal{R}^6 \to \mathcal{R}^4$ given by $T(x) = Ux$ is
ANSWER: [A] one-to-one [B] onto [C] neither one-to-one nor onto

18) Which of the statements below about the system of equations $Ux = b$ is false?
ANSWER:
[A] x_1 and x_2 are basic variables
[B] x_3 is a basic variable and x_4 is a free variable
[C] x_5 is a basic variable and x_6 is a free variable

19) The dimension of the column space of U, dimCol U, is
ANSWER: [A] 2 [B] 3 [C] 4

20) Which subset of columns of U is a basis for $Col\ U$?
ANSWER: [A] $\{1, 2, 3, 6\}$ [B] $\{1, 4, 5\}$ [C] $\{1, 2, 5\}$

21) The dimension of the orthogonal complement to the column space of U is
ANSWER: [A] 3 [B] 2 [C] 1

22) The least squares solution of the inconsistent system $Ax = b$, where
$A = \begin{bmatrix} 6 & 9 \\ 3 & 8 \\ 2 & 10 \end{bmatrix}$, and $b = \begin{bmatrix} 0 \\ 49 \\ 0 \end{bmatrix}$
ANSWER: [A] $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, [B] $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ [C] $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

23) The characteristic polynomial of A^TA, for A the matrix of problem 22 above, is:
ANSWER: [A] $84\lambda^2 + 22\lambda + 1$ [B] $\lambda^2 - 22\lambda + 84$ [C] $\lambda^2 + 22\lambda + 84$

24) The characteristic equation of a matrix M is given by $\lambda^8 - 15\lambda^6 - 2\lambda^2 = 0$. Then M must have size

ANSWER: **[A]** 2×2 **[B]** 6×6 **[C]** 8×8

25) The determinant of $\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 0 \\ 0 & 4 & 1 & 2 \\ 0 & 3 & 1 & 4 \end{bmatrix}$ is

ANSWER: [A] 6 [B] -6 [C] 16

26) The characteristic polynomial of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ is

ANSWER:
[A] $\lambda^3 - 6\lambda^2 - 3\lambda - 10$
[B] $\lambda^3 - 6\lambda^2 + 3\lambda + 10$
[C] $\lambda^3 + 6\lambda^2 - 3\lambda$

27) The matrix $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 3 & 1 & -5 & 0 \\ 1 & 2 & 0 & -5 \end{bmatrix}$ is row equivalent to the matrix $B = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. A basis for the null space of A is

ANSWER:

[A] $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ [B] $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ [C] $\left\{ \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

28) $\lambda = 1$ is an eigenvalue for the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$. The dimension of the eigenspace for $\lambda = 1$ is
ANSWER: [A] 1 [B] 2 [C] 3

29) $\lambda = -2$ is an eigenvalue for the matrix $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. A basis for the eigenspace $\lambda = -2$ is

ANSWER:
[A] $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ [B] $\left\{ \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ [C] $\left\{ \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

30) If A is an $n \times n$ diagonalizable matrix, then it has n linearly independent eigenvectors.
ANSWER: [A] True [B] False

31) If an $n \times n$ matrix A is diagonalizable, then it must have n distinct eigenvalues.
ANSWER: [A] True [B] False

32) Let A and B be 3×3 matrices such that $\det A = 3$, and $\det B = -4$. The value of $\det[(2A)^{-1}B^2B^T]$ is

ANSWER: [A] $3/8$ [B] $-8/3$ [C] $8/3$

33) Let $A = \begin{bmatrix} 5 & 0 & 0 \\ 5 & 5 & 5 \\ -2 & 0 & 3 \end{bmatrix}$. Which one of the following is the eigenspace of A associated with the eigenvalue $\lambda = 5$.

ANSWER:

[A] span $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

[B] span $\left\{ \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

[C] span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$