Name (PRINT): ______________________ Student # __________

Test #3
Mathematics 69.107
Nov 3, 2000
Full marks: 40

Answer all questions. Use non-programmable calculator if needed.

(1) [8 marks] Evaluate the following derivatives:

(a) [3 marks] $\frac{d}{dx}\int_2^{x^3} \frac{t}{\sqrt{1+t^3}}\, dt$

$$\frac{d}{dx}\int_2^{x^3} \frac{t}{\sqrt{1+t^3}}\, dt = \frac{x^3}{\sqrt{1+(x^3)^3}} \cdot \frac{d}{dx}x^3 = \frac{3x^5}{\sqrt{1+x^9}}$$

(b) [5 marks] $\frac{d}{dx}\int_{x^2}^{x^3} e^{-u^2}\, du$

$$\frac{d}{dx}\int_{x^2}^{x^3} e^{-u^2}\, du = e^{-(x^3)^2} \cdot \frac{d}{dx}x^3 - e^{-(x^2)^2} \cdot \frac{d}{dx}x^2 = 3x^2e^{-x^6} - 2xe^{-x^4}$$

(2) [8 marks]

Evaluate the area bounded by the curve

$$y = \frac{2x}{(x^2+1)^2}$$

and the lines $x = 1$, $x = 2$ and the x-axis.

$$\text{Area} \quad = \quad \int_1^2 \frac{2x}{(x^2+1)^2}\, dx = \int_2^5 \frac{1}{u^2}\, du = -\frac{1}{u}\Big|_2^5 = -\left(\frac{1}{5} - \frac{1}{2}\right) = 0.3$$

(3) [24 marks] Evaluate the following integrals and simplify the answers.

(a) [4 marks] $\int x^2(1+2x^3)^3 \; dx$

$$\int x^2(1+2x^3)^3 \; dx = \frac{1}{6}\int u^3 \; du = \frac{1}{6}\frac{u^4}{4} + C = \frac{1}{24}\left(1+2x^3\right)^4 + C$$

(b) [4 marks] $\int \sin x \; \sec^2(\cos x) \; dx$

$$\int \sin x \; \sec^2(\cos x) \; dx = -\int \sec^2 u \; du = -\tan u + C = -\tan(\cos x) + C$$

(c) [4 marks] $\int \sin x \cos x \; e^{\cos^2 x} \; dx$

$$\int \sin x \cos x \; e^{\cos^2 x} \; dx = -\frac{1}{2}\int e^u \; du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\cos^2 x} + C$$

(d) [4 marks] $\int_2^4 \frac{1+x-x^2}{x^2} \; dx$

$$\begin{aligned}\int_2^4 \frac{1+x-x^2}{x^2} \; dx &= \int_2^4 \left(x^{-2} + \frac{1}{x} - 1\right) \; dx = \left(-\frac{1}{x} + \ln x - x\right) \Big|_2^4 \\ &= \left(-\frac{1}{4} + \ln 4 - 4\right) - \left(-\frac{1}{2} + \ln 2 - 2\right) = \ln 2 - \frac{7}{4}\end{aligned}$$

(e) [4 marks] $\int_{\sqrt{\ln 3}}^{\sqrt{\ln 6}} 2x \; e^{x^2} \; dx$

$$\int_{\sqrt{\ln 3}}^{\sqrt{\ln 6}} 2x \; e^{x^2} \; dx = \int_{\ln 3}^{\ln 6} e^u \; du = e^u \; \Big|_{\ln 3}^{\ln 6} = 6 - 3 = 3$$

(f) [4 marks] $\int x \cos x \; dx$

$$\int u \; dv = uv - \int v \; du = x \sin x - \int \sin x \; dx = x \sin x + \cos x + C$$