Aperiodic Order in Dynamical Systems and Operator Algebras

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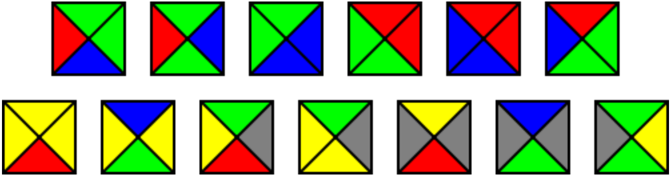
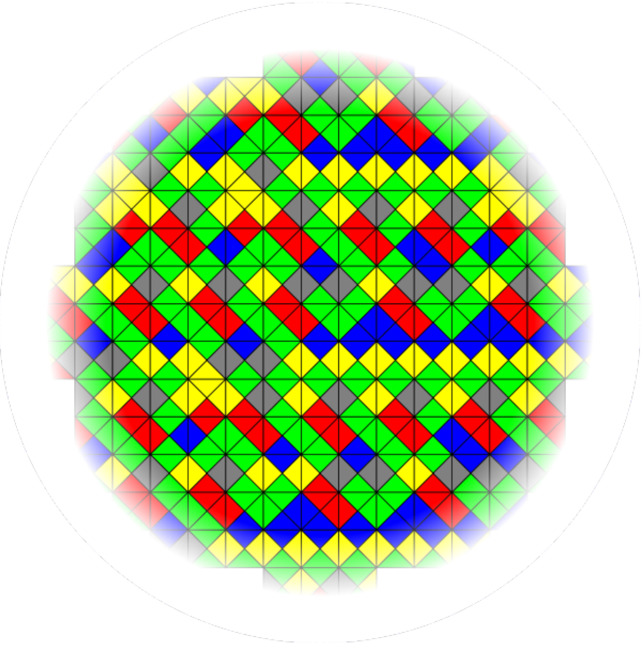
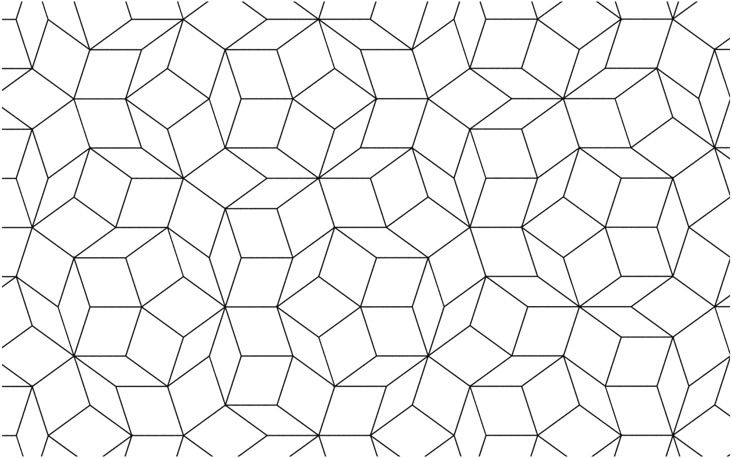
February 24, 2012

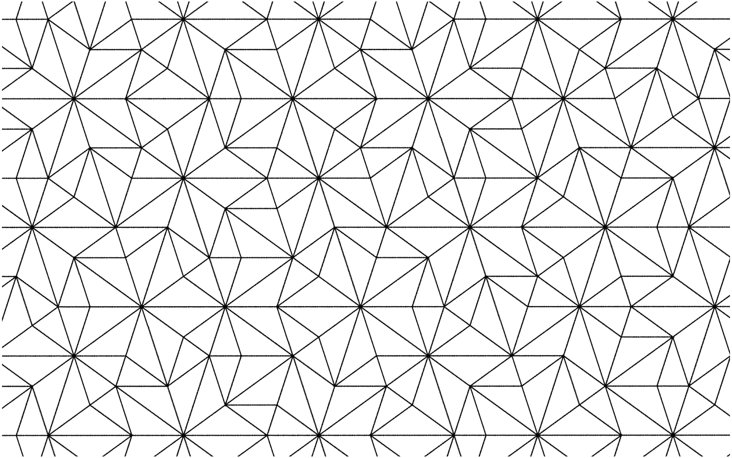


A set of “dominos”

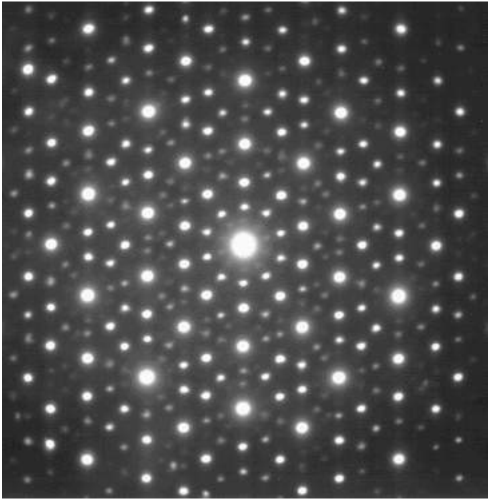




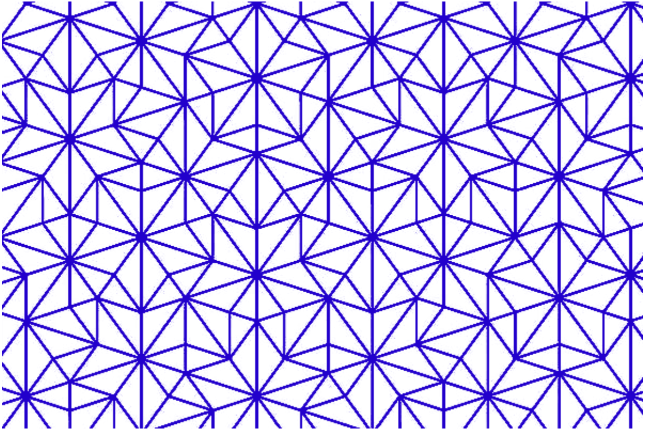
Penrose Rhomb Tiling



Penrose Triangle Tiling (Robinson Triangles)

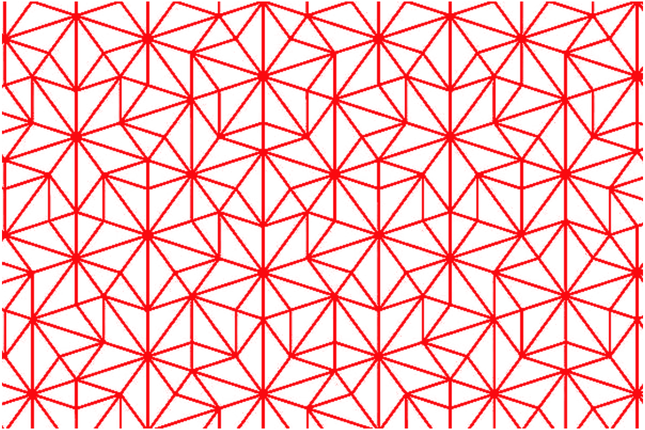


Example: Penrose Tiling



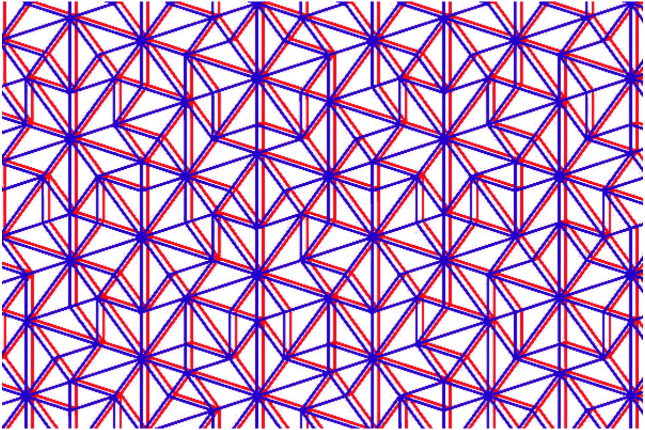
T_1

Example: Penrose Tiling



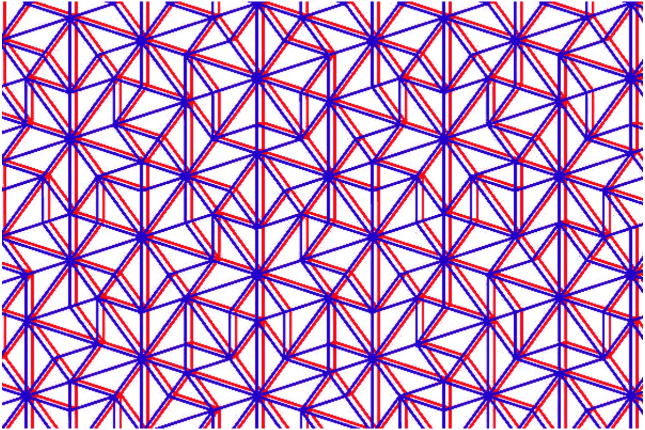
T_2

Example: Penrose Tiling



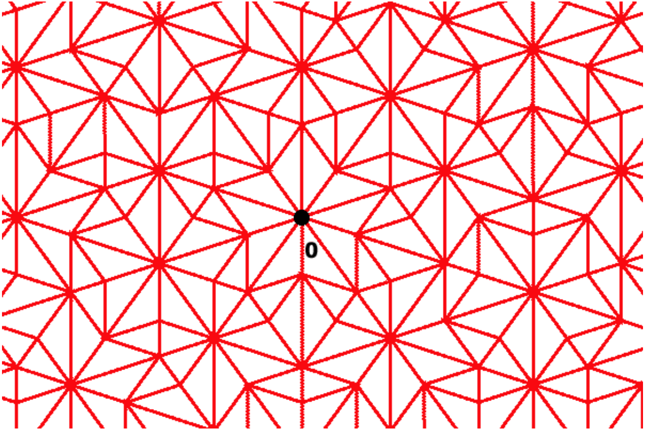
T_2 is a small shift of T_1

Example: Penrose Tiling



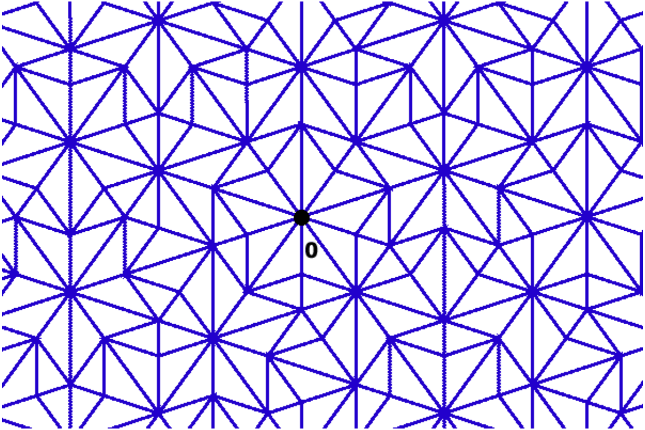
T_2 is a small shift of $T_1 \Rightarrow T_1$ is close to T_2

Example: Penrose Tiling



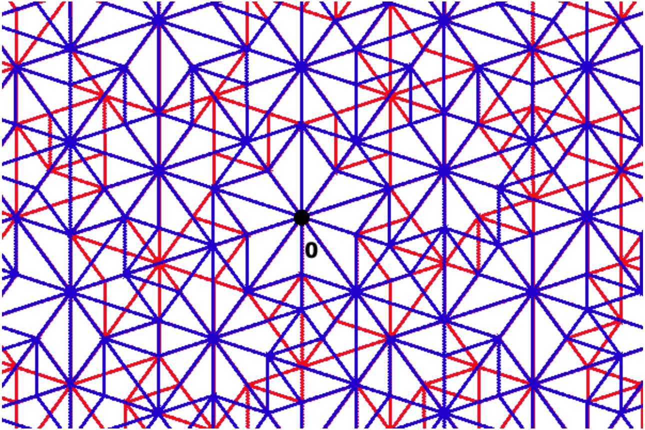
T_1

Example: Penrose Tiling



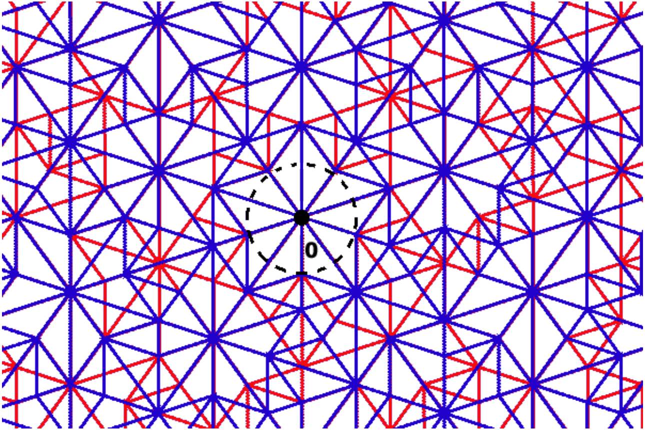
T_2

Example: Penrose Tiling



T_1 and T_2 agree around the origin, disagree elsewhere.

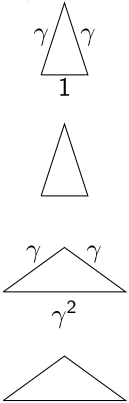
Example: Penrose Tiling



$d(T_1, T_2) <$ (radius of the ball above.)$^{-1}$

Example: Penrose Tiling

Prototiles
(+ rotates by $\frac{\pi}{5}$)
$\gamma =$ golden ratio



Example: Penrose Tiling

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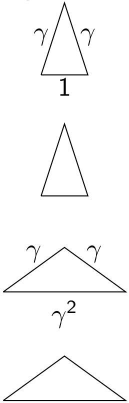

$$\overset{\omega}{\xrightarrow{\lambda=\gamma}}$$

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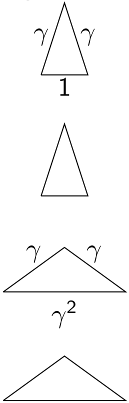

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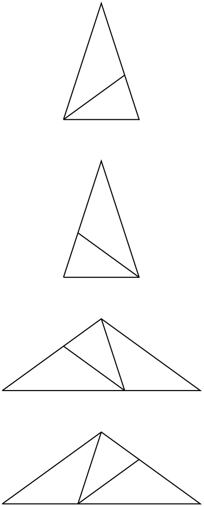

Example: Penrose Tiling

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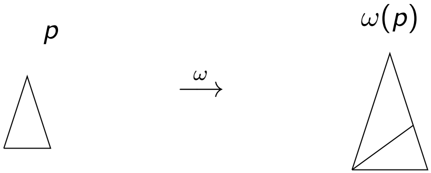
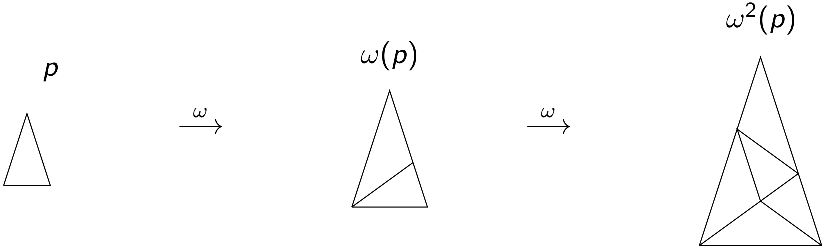
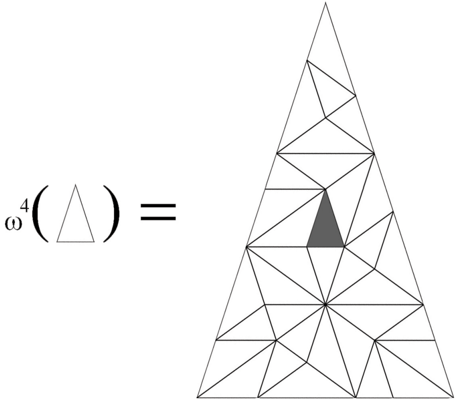
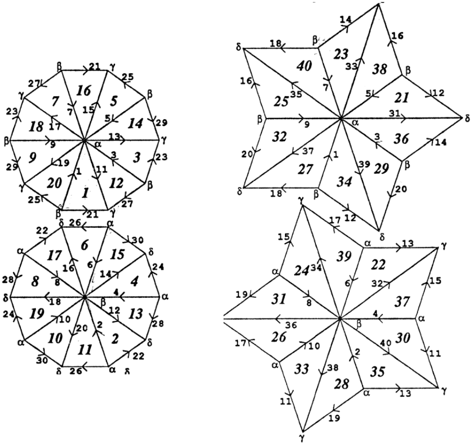


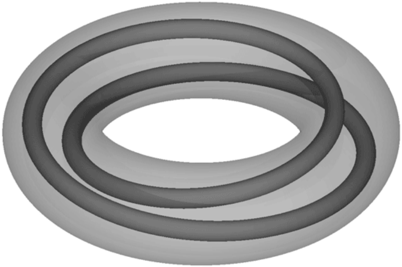
Example: Penrose Tiling

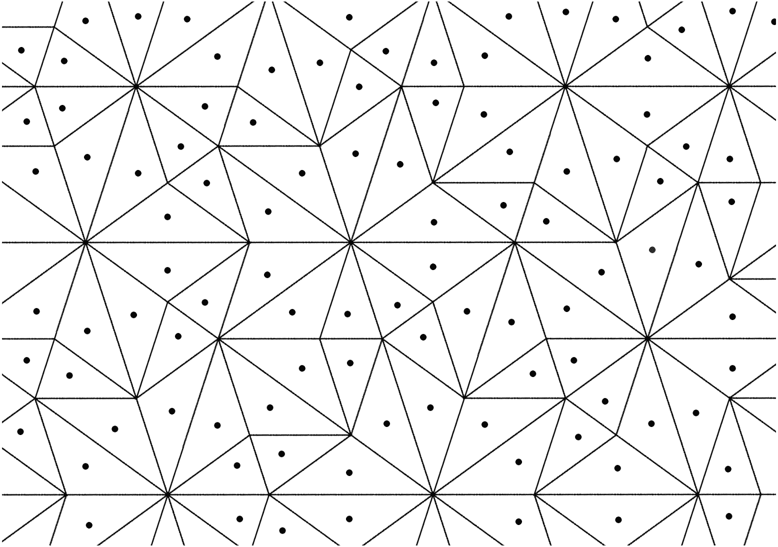


Producing a Tiling from a Substitution Rule

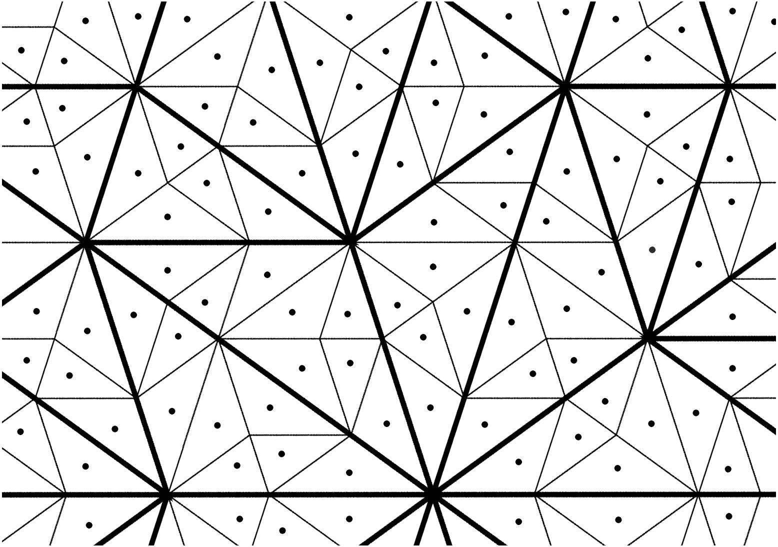



16 5 14 3 12 1 20 9 18 7
6 15 4 13 2 11 10 19 8 17
23 38 21 36 29 34 27 32 25 40
39 22 37 30 35 28 33 26 31 24

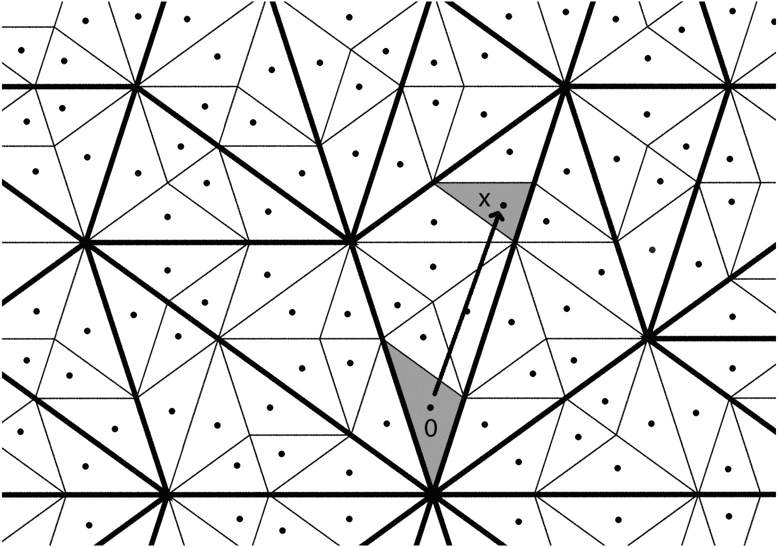




A punctured tiling T.



T has unique decomposition into 2nd order supertiles.



$$(T, T - x) \in \mathcal{R}_2.$$