Seventh Annual Ottawa-Carleton Applied Analysis Day

Friday November 8, 2024, Carleton University, Ottawa

Objective: This one-day workshop brings together researchers from the Ottawa-Carleton Applied Analysis Group and from other departments, as well as from the wider community at nearby institutions. The event features keynote lectures by Prof. Katrin Rohlf of Toronto Metropolitan University and Prof. Jean-Christophe Nave from McGill University. It also includes short talks by local faculty members, graduate students and postdoctoral fellows on topics in applied analysis, differential equations, numerical analysis and applications in a variety of areas.

Location: 4351 HP (Herzberg Laboratories)

SCHEDULE

8:30 – 9:00	WELCOME and COFFEE
9:00 – 10:00	**Katrin Rohlf** Particle-based methods for biological applications
10:00 – 10:25	**Archishman Saha** Moser Transformation of a Stochastically Perturbed Kepler Problem
10:25 – 10:45	COFFEE BREAK
10:45 – 11:10	**Emmanuel Lorin** Polarization learning in attosecond science
11:10 – 11:35	**Sumiya Baasandorj** $C^{1,\alpha}$-regularity for entropic optimal transport problems
11:35 – 12:00	**Justin Harry Martel** Introducing Weber's Electrodynamics and His Surprising -2 Electric Molecules
12:00 – 1:30	LUNCH BREAK
1:30 – 2:30	**Jean-Christophe Nave** Functional discretization of problems with a wide range of scales
2:30 – 2:55	**Anne Marie Conway** Goal-oriented Error Estimation: Comparison of different error representations in adaptive algorithms
2:55 – 3:15	COFFEE BREAK
3:15 – 3:40	**Stacey Smith?** Key factors and parameter ranges for immune control of equine infectious anemia virus infection
3:40 – 4:05	**Lin Wang** Effects of heterogeneous distribution of resources and toxins on movement strategies

ABSTRACTS

Katrin Rohlf
Department of Mathematics, Toronto Metropolitan University
Title: **Particle-based methods for biological applications**
Abstract: Interest in using and developing particle-based methods for biological applications continues to be an area of interest, and has grown in popularity with the advancement of computer technology. Their higher computational demand continues to be a challenge for some applications, such as for blood flow in biologically meaningful flow geometries. However, their ability to capture and explore phenomena that are difficult or challenging to model using traditional differential equations, such as the Navier-Stokes equations for fluid flow for example, provides motivation for their continued development.

A numerically efficient particle-based method called RMPC (Reactive Multi-Particle Collision) dynamics will be introduced, and a number of biologically-motivated applications will be discussed. Examples will include recent advancements in using non-reactive MPC (RMPC without reactions) in weakly, compressible flow through a local constriction, as well as diffusion-control challenges and remedies for simulations of highly spatially inhomogeneous diffusion problems. Recent progress using RMPC dynamics (MPC with added reactions) for stochastic self-assembly, as well as reinfection statistics for an epidemic model, will be presented.

Jean-Christophe Nave
Department of Mathematics and Statistics, McGill University
Title: Functional discretization of problems with a wide range of scales
Abstract: In this talk I will present a new strategy to solve evolution PDEs which solutions develop a wide range of scales. Typical approaches to capture fine scales include mesh refinement (h-adaptivity) and polynomial adaptivity. I will show that by geometrizing the problem, one may leverage the semigroup structure of the solution operator. As a result, one may discretize this diffeomorphism as a composition of submaps over finite time intervals. This leads to a new type of space-time adaptivity: compositional adaptivity. This approach possesses exceptional resolution properties while remaining computationally efficient. I will illustrate this technique with problems from turbulence.

Archishman Saha
Department of Mathematics and Statistics, University of Ottawa
Title: Moser Transformation of a Stochastically Perturbed Kepler Problem
Abstract: We consider a stochastic Kepler problem perturbed by a Hamiltonian noise affecting the angular momentum vector. We show that while the angular momentum and the Laplace-Runge-Lenz vectors are not conserved, their norms satisfy the usual deterministic dynamics. This allows us to determine the set of initial conditions leading to collisions. Further, in a procedure similar to Moser regularization, we transform the stochastic Kepler problem to obtain its dynamics as a stochastic geodesic flow on a 3-sphere.

Emmanuel Lorin
School of Mathematics and Statistics, Carleton University
Title: Polarization learning in attosecond science
*Abstract:*The primary objective of this talk is to explore key mathematical and computational aspects in attosecond science. I will then demonstrate how supervised learning can efficiently predict the emission of high-frequency photons from laser-molecule interaction. If time permits, I will discuss applications in nonperturbative nonlinear optics.

Sumiya Baasandorj
Department of Mathematics and Statistics, Univeristy of Ottawa
Title: $C^{1,\alpha}$-regularity for entropic optimal transport problems
Abstract: In this talk, we discuss the stability of $C^{1,\alpha}$-regularity for entropic optimal transport problems and its connection to the Monge-Amp‘ere equation. This is a recent joint work with Simone Di Marino and Augusto Gerolin.

Justin Harry Martel
Title: Introducing Weber's Electrodynamics and His Surprising -2 Electric Molecules
Abstract: In this brief talk we introduce the electrodynamics of the German physicist Wilhelm Weber (1804-1891) whose collected works have recently been translated into English. Weber's electrodynamics has many surprising predictions including Weber's interpretation of $E = \mu c^2$, effective inertial mass, and stable $[-1, -1] = [-2]$ electric molecules.

About the Speaker: Justin Harry Martel completed his PhD at University of Toronto in 2019 under Prof. R. J. McCann, with a thesis in optimal transport and algebraic topology titled ”How to Build Spines from Singularity”. JHM continues to work in Weberian electrodynamics, optimal transport, and computational geometry.

Anne Marie Conway
Department of Mathematics and Statistics, University of Ottawa
*Title:*Goal-oriented Error Estimation: Comparison of different error representations in adaptive algorithms
Abstract: In finite element analysis, many error estimation techniques focus on quantifying the approximation error in a global norm, namely the error on the entire domain. This can be sub-optimal for applications requiring a controlled error only in some parts of the domain or in a specific quantity, depending on the solution. Goal-oriented error estimation focuses on estimating the approximation error of a specific quantity of interest.

This talk presents several globally equivalent error representations that are used to drive goal-oriented adaptive algorithms. The error estimates are defined in the context of a well-posed abstract problem, using duality techniques and Galerkin orthogonality properties. They are then applied within h-adaptive algorithms based on different marking strategies, considering a 1D boundary-layer problem and the 2D Poisson problem on an L-shaped domain. Numerical results demonstrate how the error representations vary locally, and how the choice of the representation affects the performance of the adaptive algorithms. The results are then used to determine a ranking, in terms of efficiency and accuracy, of the various error representations.

Stacey Smith?
Department of Mathematics and Statistics, University of Ottawa
Title: Key factors and parameter ranges for immune control of equine infectious anemia virus infection
Abstract: Equine Infectious Anemia Virus (EIAV) is an important infection in equids, and its similarity to HIV creates hope for a potential vaccine. We analyze a within-host model of EIAV infection with antibody and cytotoxic T lymphocyte (CTL) responses. In this model, the stability of the biologically relevant endemic equilibrium, characterized by the coexistence of long-term antibody and CTL levels, relies upon a balance between CTL and antibody growth rates, which is needed to ensure persistent CTL levels. We determine the model parameter ranges at which CTL and antibody proliferation rates are simultaneously most influential in leading the system towards coexistence and can be used to derive a mathematical relationship between CTL and antibody production rates to explore the bifurcation curve that leads to coexistence. We employ Latin hypercube sampling and least squares to find the parameter ranges that equally divide the endemic and boundary equilibria. We then examine this relationship numerically via a local sensitivity analysis of the parameters. Our analysis is consistent with previous results showing that an intervention (such as a vaccine) intended to control a persistent viral infection with both immune responses should moderate the antibody response to allow for stimulation of the CTL response. Finally, we show that the CTL production rate can entirely determine the long-term outcome, regardless of the effect of other parameters, and we provide the conditions for this result in terms of the identified ranges for all model parameters.

Lin Wang
Department of Mathematics and Statistics, University of Ottawa
Title: Effects of heterogeneous distribution of resources and toxins on movement strategies
Abstract: In this talk, we explore optimal movement strategies for organisms in landscapes characterized by spatially varying distributions of resources and toxins. Our investigation is inspired by recent experiments on C. elegans movement in environments with heterogeneously distributed food and toxins. The authors found that when resources and toxins overlap spatially, fast diffusion was beneficial for the population, whereas when they were in different locations, slow diffusion was better. We model the experimental system using reaction-diffusion-advection equations with a stage-structured population. We analyze how the principal eigenvalue of the linearized system depends on the diffusion and advection parameters, to identify the best combination between random motion (seeking resources) and directed motion (moving away from toxins, avoiding danger). Our main result is to determine the conditions under which the principal eigenvalue is monotonic with respect to diffusion or advection parameters, which will clarify whether larger or smaller diffusion or advection are beneficial. Specifically, we identify limiting scenarios in which diffusion and advection are very small or large. These theoretical insights deepen our comprehension of optimal movement strategies in heterogeneous environments and provide a theoretical basis for interpreting experimental observations.