Third Annual Ottawa-Carleton Applied Analysis Day

October 11, 2019, Carleton University, Ottawa

**Objective:** This one-day workshop brings together researchers from the Ottawa-Carleton Applied Analysis Group and from other departments, as well as from the wider community at nearby institutions. The event features a keynote lecture by Prof. Nilima Nigam of Simon Fraser University, as well as short talks by local faculty members, graduate students and postdoctoral fellows on topics in applied analysis, differential equations, numerical analysis and applications in a variety of areas.

### SCHEDULE

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Nilima Nigam  
Department of Mathematics, Simon Fraser University  
*Title:* Mixed Dirichlet-Neumann eigenvalues: computation and applications  
*Abstract:* Eigenvalue problems with mixed boundary conditions - with Dirichlet conditions on some parts of the boundary, and Neumann on others - arise in many applications. The eigenfunctions of such problems can possess low regularity and are challenging to compute using standard techniques. We present a high-accuracy numerical algorithm for the mixed Dirichlet-Neumann eigenproblems of the Laplacian based on a boundary integral formulation, and then present its use in applications arising in spectral geometry and signal processing.

Brandon Robinson  
Department of Civil and Environmental Engineering, Carleton University  
D. Poirel, Department of Mechanical and Aerospace Engineering, Royal Military College of Canada, C. Pettit, Aerospace Engineering Department, United States Naval Academy, M. Khalil, Sandia National Laboratories, A. Sarkar, Department of Civil and Environmental Engineering, Carleton University  
*Title:* Numerical solution of a coupled PDE and ODE system in nonlinear aerelasticity  
*Abstract:* A system of nonlinear coupled partial differential equations and ordinary differential equation is derived to describe the motion of an elastically mounted flexible airfoil which mimics the wind-tunnel experimental setup at the Royal Military College of Canada. The system consists of a flexible airfoil which is coupled with a rigid body rotational motion at the base. The dynamics of the system is described by a coupled set of nonlinear partial differential equations (PDEs) which govern the elastic vibrations of the airfoil and an ordinary differential equation (ODE) that describes the rigid body rotation at the base. The coupling of a flexible airfoil with a rigid body base rotation introduces inertial nonlinearities, which are atypical in structural dynamics. Consequently the equations of motion cannot be easily converted to a state-space form, and an implicit solution scheme is adopted. Furthermore, upon discretizing the PDEs using a Galerkin method, an efficient solution is devised which is then verified by using a finite difference scheme based on the Hubolt method. In order to effectively use the finite difference scheme, the nonlinear PDE and ODE system is tackled in a manner analogous to the conservative form of Burgers equation.

Maryam Basiri  
Department of Mathematics and Statistics, University of Ottawa  
Abbas Moameni, School of Mathematics and Statistics, Carleton University, Frithjof Lutscher, Department of Mathematics and Statistics, University of Ottawa  
*Title:* Pushing Boundaries: The existence of solutions for a free boundary problem modeling the spread of ecosystem engineers  
*Abstract:* Modeling the movement and spread of invasive species into a new environment has been the subject of many publications in population ecology. The overwhelming majority of models for spreading populations are based on Fisher’s reaction-diffusion equation. This approach assumes that the habitat quality is independent of the population. Ecosystem engineers are species that modify their environment in order to make it (more) suitable for them. Beavers are a well-known engineering species. A potentially more suitable modeling approach in this case is to adapt the well-known Stefan problem of melting ice. Ahead of the front, the habitat is unsuitable for the species (the ice); behind the front, the habitat is suitable (the open water). The engineering action of the population moves the boundary ahead (the melting). This modeling approach leads to a time-dependent free boundary problem where the boundary corresponds to the edge of the population front. In this talk, we present a novel model for the spread of ecosystem engineers as a free boundary problem. We derive the semilinear parabolic equation from an individual random walk model. The Stefan condition for the moving boundary is replaced by a biologically derived two-sided condition that models the movement behavior of individuals at the boundary as well as the process by which the population moves the boundary to expand their territory. We then prove local and global existence and uniqueness of solutions to the equations. We assign a convex functional to this problem, so that the evolution system governed by this convex potential is exactly the system of evolution equations describing the above model. We shall then apply variety of variational, fixed point and other methods to deal with this free boundary problem.

Aliou Sow  
Department of Mechanical Engineering, University of Ottawa  
*Title:* Shock Bifurcation in Cellular Detonations: A Purely gasdynamic Driven Mechanism?  
*Abstract:* During the cellular dynamics of gaseous detonation waves, upon collisions of triple shocks, forward jets can be triggered under certain conditions. The forward jets can interact with the Mach stem causing it to bifurcate. Shock bifurcation is a mechanism deemed to be responsible of cell irregularity. The present study aim to unveil the mechanism of shock bifurcation in cellular gaseous detonations. Two-dimensional numerical simulations are performed. To ease the clarification of the role of gasdynamics, a simple Arrhenius kinetic law is used as for the chemical model. The acti-
viation energy with respect to the shock state is kept constant to obtain the same reaction rate sensitivity to temperature in all considered mixtures. This procedure allow to dissociate the gasdynamics effects from the chemistry effects. The comparison between the detonation results and the inert shock reflection model predictions indicates that the shock bifurcation process is primarily a gasdynamic driven mechanism favoured by high Mach numbers and low specific heat ratio.

Rimple Sandhu
Department of Civil and Environmental Engineering, Carleton University
M. Khalil, Sandia National Laboratories, C. Pettit, Aerospace Engineering Department, United States Naval Academy, D. Poiriel, Department of Mechanical and Aerospace Engineering, Royal Military College, A. Sarkar, Department of Civil and Environmental Engineering, Carleton University
Title: Sparse probabilistic learning of nonlinear differential equations
Abstract: Predictive modeling of nonlinear engineering systems with an incompletely understood physics and available field measurements is addressed in this work. Here, nonlinearity implies a nonlinear relation between the unknown model parameters and the measured entity. For such systems, an overly-parametrized stochastic differential equation (SDE) is usually proposed to quantify the intended physics by the careful addition of purely statistical elements in the existing laws-of-physics equations. The Bayesian framework is well-suited to estimate the posterior probability density function (pdf) of unknown parameters by assimilating noisy measurements with the proposed SDE. Sparse learning of an overly-parametrized system becomes crucial when calibrating overly-parametrized SDEs to alleviate overfitting and improve generalization of the calibrated model. Sparse learning of nonlinear SDEs is tackled in this work using a semi-analytical Bayesian framework that exploits Gaussian automatic relevance determination (ARD) priors, leading to a predictive model nested under the proposed SDE that provides an optimal trade-off between complexity and data-fit (as facilitated by Bayesian model evidence). ARD priors have zero-mean and an unknown precision (called hyper-parameter) that dictates the contribution of the corresponding parameter in model predictions. During maximization of Bayesian model evidence, ARD priors of superficial parameters approach Dirac-delta functions centered at zero, effectively pruning them off from the model. This removal of redundant parameters from SDEs leads to an optimal SDE that only includes parameters that are relevant to the intended physics, and is less prone to overfitting. The mathematical formulation and application of the proposed sparse learning framework will be discussed in detail during the talk.

Matei Radulescu
Department of Mechanical Engineering, University of Ottawa
Title: Nonlinear dynamics of detonations using Fickett’s model
Abstract: The present talk discusses recent results of detonation dynamics obtained using Fickett’s toy model for reactive gasdynamics. The model consists of a reactive form of the inviscid Burgers’ equation, which couples the forward facing pressure waves with the reactivity field. Rational derivations from the reactive Euler equations are discussed. Using the simple model, we illustrate the instability mechanism of detonation waves and its non-linear dynamics transition to chaos via period doubling. An oscillator model composed of a third order differential equation is constructed by asymptotic methods from the Fickett model by a multiple time scale method, which is found to recover the non-linear dynamics. The source of period doubling bifurcations is discussed with the Fickett model in the singular limit of galloping detonations with pulsed energy release.

Saint-Cyr E.R. Koyaguerebo-Imé and Yves Bourgault
Department of Mathematics and Statistics, University of Ottawa
Title: Arbitrary high-order time-stepping methods for reaction diffusion equations via deferred correction
Abstract: The space-discretization of time-evolution partial differential equations usually lead to stiff initial value problems (IVP) of large dimension. To avoid overly small time steps, accurate approximate solutions for these IVP are obtained with high-order time-stepping methods with satisfactory stability properties (A-stable method are of great interest). Backward differentiation formulae (BDF) of order 1 and 2 are commonly used according to their A-stability property, but BDF methods of order 3 and higher lack stability properties (e.g. for systems with complex eigenvalues). We propose an approach based on deferred corrections inspired by [Gustafsson and Kress, 2012] for the time discretization of these problems. The method is based on a successive correction (perturbation) of the implicit midpoint rule, increasing the order of accuracy by two per correction step and keeping the A-stable property of the trapezoidal rule for each level of the correction. It results an unconditionally stable methods of arbitrary high order in time when applied to nonlinear reaction-diffusion equations discretized by finite element methods in space. A complete numerical analysis of the method was done with stability and error estimates using a new Deferred Correction Condition (DCC). A numerical illustration using the bi-stable reaction-diffusion equation with the schemes of order 2, 4, 6, 8 and 10 confirms the order of the method.

P. V. Sudhi
Department of Civil and Environmental Engineering, Carleton University
A. Desai, External Collaborator, Carleton University, M. Khalil, Sandia National Laboratories, C. Pettit, Aerospace En-
Abstract: Landscape fragmentation arises from human activities and natural causes, and may create abrupt transitions (‘interfaces’) in landscape quality. How landscape fragmentation affects ecosystems diversity and stability depends, among other things, on how individuals move through the landscape. In this work, we focus on the movement behavior at an interface between habitat patches of different quality. Specifically, we study how this individual-level behavior affects the steady state of a density of a diffusing and logistically growing population in two adjacent patches. We consider a model for population dynamics in a habitat consisting of two homogeneous one-dimensional patches in a coupled ecological reaction diffusion equation. The movement between patches is incorporated into the interface conditions. We establish the existence, uniqueness, and global asymptotic stability of the steady state. Then we explore how the qualitative properties of the steady state depend on movement behavior. We apply our analysis to a previous result where it was shown that a randomly diffusing population in a continuously varying habitat can exceed the carrying capacity at steady state. We clarify the role of nonrandom movement in this context. In particular, we determine conditions on movement rates and patch preference, so that the steady-state density exceeds the carrying capacity.

Emmanuel Lorin
School of Mathematics and Statistics, Carleton University
Title: Double-preconditioning for fractional linear systems
Abstract: This work is devoted to the numerical computation of solutions to “fractional linear systems” (FLS), \( A^α x = b \), for \( α \in \mathbb{R}_+ \) and where \( A \) is a large (sparse) matrix. Cauchy integral and differential-based preconditioners will be introduced for efficiently solving FLS. Joint work with X. Antoine (IECL, Nancy)

She-Ming Lau-Chapdelaine
Department of Mechanical Engineering, University of Ottawa
Title: Detonation model using the Burgers equation and a pulsed reaction
Abstract: This study uses a simplified detonation model to investigate the behaviour of detonations with galloping-like pulsations. Burgers equation is used for the hydrodynamic equation, coupled to a source term to account for reactions. The source term is pulsed, whereby all the shocked reactants are simultaneously consumed at fixed intervals. No reactions occur in the mean time, and the interface between burnt and unburnt gasses remains still in the lab frame of reference. This model mimics the short periodic amplifications of the shock front followed by relatively lengthy decays seen in galloping detonations. The Burgers equation displays only forward-travelling characteristics and the sudden reaction of all shocked material simplifies the complicated dynamics of re-amplification. Numerical simulations performed in the shock attached frame reveal a sawtooth evolution of the front velocity with a period-averaged front speed equal to the Chapman-Jouget detonation velocity. The front velocity is composed of two distinct decay rates, punctuated by reactions. The same progression is reflected in the density profile. A strong expansion wave is created at the last location of reaction interface after each pulse. A characteristic investigation reveals that characteristics originating from the head of this expansion take approximately one-and-a-half periods to reach and attenuate the detonation front, while characteristics from the tail take up to two-and-a-half periods, for the parameters tested. The leading characteristics are amplified twice by passing through subsequent reaction interfaces, before arriving at the front, whilst the weaker trailing characteristics are amplified three times. This leads to the two distinct decay rates seen in the detonation front speed.

Nazanin Zaker
Department of Mathematics and Statistics, University of Ottawa
Title: The effect of movement behavior on population density in fragmented landscapes
Abstract: Landscape fragmentation arises from human activities and natural causes, and may create abrupt transitions (‘interfaces’) in landscape quality. How landscape fragmentation affects ecosystems diversity and stability depends, among other things, on how individuals move through the landscape. In this work, we focus on the movement behavior at an interface between habitat patches of different quality. Specifically, we study how this individual-level behavior affects the steady state of a density of a diffusing and logistically growing population in two adjacent patches. We consider a model for population dynamics in a habitat consisting of two homogeneous one-dimensional patches in a coupled ecological reaction diffusion equation. The movement between patches is incorporated into the interface conditions. We establish the existence, uniqueness, and global asymptotic stability of the steady state. Then we explore how the qualitative properties of the steady state depend on movement behavior. We apply our analysis to a previous result where it was shown that a randomly diffusing population in a continuously varying habitat can exceed the carrying capacity at steady state. We clarify the role of nonrandom movement in this context. In particular, we determine conditions on movement rates and patch preference, so that the steady-state density exceeds the carrying capacity.