

Homework 2
MATH4805/COMP4805/MATH5605 Theory of Automata
Fall 2011 – Due on 1 December 2011

Students in MATH4805 or COMP 4805 answer all questions except 4 and 10. MATH5605 students answer all questions.

- (1) Minimize the following DFA:

	a	b
$\rightarrow s_0$	s_1	s_2
s_1	s_2	s_3
s_2	s_2	s_3
$\leftarrow s_3$	s_3	s_4
s_4	s_2	s_3

- (2) Find the Minimal automaton recognizing $L = 1(0^*1)^*1 + \epsilon$. For this language, give a monoid, M , a homomorphism

$$f : \{0, 1\}^* \rightarrow M$$

and a set $F \subseteq M$ such that $L = f^{-1}(F)$.

- (3) The language $L = (0 + 1)^*1(0 + 1)^2$ can be recognized by a 4 state NFA and an 8 state DFA. Give the Transition monoids of each. The transition monoid of the DFA will be a submonoid of a row monomial monoid. The transition monoid of the NFA will be a submonoid of the $n \times n$ matrices over the Boolean algebra. What is the unique (up to isomorphism) minimal monoid that recognizes this language?

- (4) (**bonus for MATH/COMP 4805**) Give the minimal DFA and the minimal monoid that recognize the content determined language

$$\mathcal{L} = \{w = \{a, b, c\}^* \mid w \text{ contains } a \text{ or } c \text{ but not both unless it also contains an } b\}$$

Verify that the monoid is idempotent and commutative.

- (5) Let $L = \{a^i b^j c^k \mid i > j > k\}$. Prove that L is not context-free.
- (6) Find a PDA recognizing the language generated from the following grammar:

$$\begin{aligned} S &\rightarrow aTb \mid bTa \mid TT \\ T &\rightarrow bTb \mid aTa \mid \epsilon \end{aligned}$$

- (7) Give a context-free grammar for the language

$$\{w \in \{a, b\}^* \mid w \text{ has even length and its middle two symbols are "ba"}\}.$$

(8) L_1 and L_2 are context-free languages. Prove or Disprove each of the following:

- (i) $L_1 \cap L_2$ is context-free
- (ii) $L_1 + L_2$ is context-free
- (iii) $L_1 L_2$ is context-free
- (iv) L_1^* is context-free
- (v) L_1^c is context-free.

(9) Consider the following two languages

$$\mathcal{L}_1 = \{w \in \{0, 1, 2\}^* | w = 0^i 1^{2^i} 2^i\}$$
$$\mathcal{L}_2 = \{w \in \{0, 1, 2\}^* | w = 0^i 1^j 2^k \text{ and } k = 2i + j\}$$

- (i) Prove that one of the languages is Context free by providing a Context Free Grammar for it
- (ii) Prove that the other language is not Context free by using the appropriate Pumping Lemma.

(10) (**bonus for MATH/COMP 4805**) Give a PDA and a context-free grammar recognizing the languages $L_1 = \{a^m b^n c^n | m, n \geq 0\}$ and $L_2 = \{a^i b^j | i \geq j\}$.