

Exercises: Sequences and Series

In the following exercises you may find it useful to remember the result that states that: If

$$\lim_{n \rightarrow \infty} |a_n| = 0$$

then

$$\lim_{n \rightarrow \infty} a_n = 0.$$

Find the following limits if they exist... If not, explain why not:

1. $\lim_{n \rightarrow \infty} \frac{(-1)^n}{n^2}$. Ans: 0.

2. $\lim_{m \rightarrow \infty} \frac{(-1)^m}{m^3} \cos(m)$. Ans: 0.

3. $\lim_{m \rightarrow \infty} \frac{m^2 + 2}{3m^2 + 4}$. Ans: 1/3.

4. $\lim_{n \rightarrow \infty} (-1)^n \left(1 + \frac{3}{n}\right)$. Ans: Does not exist.

5. $\lim_{n \rightarrow \infty} n^2 e^{-n}$. Ans: 0.

Determine whether the given series converges to a finite value, an infinite value or diverges. Give reasons:

6. $\sum_{n=1}^{\infty} (-1)^n \frac{n^4 + 1}{2n^4 + 3}$ Ans: Diverges

7. $\sum_{n=0}^{\infty} \frac{6}{n+3}$ Ans: Converges to $+\infty$.

8. $\sum_{n=0}^{\infty} \frac{3}{n^2 + 1}$ Ans: Converges to a finite value

9. $\sum_{n=0}^{58} 3^n$ Ans: $\frac{3^{59} - 1}{2}$.

10. $\sum_{m=0}^{\infty} \frac{2}{(m+2)(m+3)}$ Ans: 1

11. $\sum_{m=0}^{\infty} (\text{Arctan}(m+1) - \text{Arctan}(m))$ Ans: $\frac{\pi}{2}$.

12. $\sum_{n=0}^{\infty} \frac{n}{(n+2)(n+3)}$ Ans: Diverges