

Miscellaneous Exercises on Series

In the following exercises determine the convergence, divergence or absolute convergence of the given series using any Test and give reasons.

1. $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ Answer: Converges to a finite value by the Ratio Test
2. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{n!}$ Answer: Absolutely convergent (and so convergent to a finite value) by the Ratio Test
3. $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$ Answer: Converges to a finite value by the Ratio Test
4. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ Answer: Converges to infinity by the Ratio Test
5. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{3^n}$ Answer: Absolutely convergent (and so convergent to a finite value) by the Ratio Test
6. $\sum_{n=0}^{\infty} \frac{(n+1)!}{2^n n!}$ Answer: Converges to a finite value by the Ratio Test
7. $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$ Answer: Converges to a finite value by the Ratio Test
8. $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{6}{n(\log n)^2}$ Answer: Absolutely convergent (and so convergent to a finite value) by the Integral Test
9. $\sum_{n=1}^{\infty} \frac{4n}{1+n^2}$ Answer: Converges to infinity by the Comparison Test
10. $\sum_{n=1}^{\infty} \frac{2^n}{n 3^n}$ Answer: Converges by comparison with a geometric series with $r = 2/3$.