

**Multiple-Choice Questions**  
**Please circle only one answer.**

1. [2 marks] Let  $f(x) = x^3 + \sqrt{x}$ . Evaluate  $f'(1)$ . In other words, find the derivative of  $f$  at  $x = 1$ .

- (a)  $f'(1) = 0$
- (b)  $f'(1) = 1.5$
- (c)  $f'(1) = 3.5$
- (d)  $f'(1) = 6$

2. [2 marks] Let  $f(x) = \ln(\sqrt{x})$ . Evaluate  $f''(2)$ . In other words, find the second derivative of  $f$  at  $x = 2$ .

- (a)  $f''(2) = -\frac{1}{8}$
- (b)  $f''(2) = 0$
- (c)  $f''(2) = -1$
- (d)  $f''(2) = \frac{3}{4}$

3. [2 marks] Let  $f(x) = |x - 1|$ . Calculate

$$L = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}.$$

- (a)  $L = 0$
- (b) This limit does not exist
- (c)  $L = -1$
- (d)  $L = 1$

4. [2 marks] Evaluate the following limit:  $L = \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$ .

- (a)  $L = 1$
- (b)  $L = 4$
- (c)  $L = 2$
- (d)  $L = -1$

5. [2 marks] A differentiable function  $f$  has the property that  $f(1) = 6$ ,  $f'(6) = -2$  and  $f'(1) = 3$ . What is the value of the derivative of  $f(f(x))$  at  $x = 1$ ?

- (a)  $-3$
- (b)  $-6$
- (c)  $x$
- (d)  $2$

6. [2 marks] Let  $f(x) = \tan(1 + \sin x)$ . Evaluate  $f'(x)$ . In other words, find the derivative of  $f$  at  $x$ .

- (a)  $f'(x) = -\sin x \sec^2(\cos x)$
- (b)  $f'(x) = \sec^2(1 + \sin x)$
- (c)  $f'(x) = \sec^2(\sin x)$
- (d)  $f'(x) = \cos x \sec^2(1 + \sin x)$

7. [2 marks] Let  $y$  be given implicitly as a differentiable function of  $x$  by  $x^2 \cos y + y^2 - 1 = 0$ . Then the slope of the tangent line to the curve  $y = y(x)$  at the point  $(x, y)$  where  $x = 0$ ,  $y = 1$  is equal to:

- (a) 2,
- (b)  $+\infty$ ,
- (c)  $\frac{1}{2}$ ,
- (d) 0.

8. [2 marks] Let  $f(x) = (x + 1)^{3x}$ . Evaluate  $f'(0)$ . In other words, find the derivative of  $f$  at  $x = 0$ .

- (a)  $f'(0) = 0$
- (b)  $f'(0) = 1$
- (c)  $f'(0) = -1$
- (d)  $f'(0) = 12$

9. [2 marks] Let  $f(x) = \text{Arcsin}(\cos x^2)$ . Calculate  $f'(x)$  in the case where  $x$  is a point where  $\sin(x^2) > 0$ .

- (a)  $f'(x) = -5$
- (b)  $f'(x) = x$
- (c)  $f'(x) = -2x$
- (d) None of these

10. [2 marks] Evaluate  $L = \lim_{x \rightarrow 0} \left( \frac{\sin 4x}{\sin 2x} \right)$  using any method.

- (a)  $L = 1$
- (b)  $L = 1.85$
- (c)  $L = 2$
- (d)  $L = \frac{5}{2}$

11. [2 marks] Evaluate

$$\lim_{x \rightarrow +\infty} \frac{d}{dx} \int_{\sqrt{3}}^{\sqrt{x}} \frac{r^3}{(r+1)(r-1)} dr$$

- (a)  $I = 0$
- (b)  $I = \frac{27}{2}$
- (c) This limit does not exist
- (d)  $I = \frac{1}{2}$
12. [2 marks] Which of the following functions  $F$  represents the form of the inverse function of the function  $f(x) = \sqrt{x^2 - 4}$  whose domain is the set of all real numbers  $x$  where  $x \geq 2$ .

- (a)  $F(x) = \sqrt{x^2 + 4}$  where  $Dom(F) = \{x : 0 \leq x < +\infty\}$
- (b)  $F(x) = -\sqrt{x^2 + 4}$  where  $Dom(F) = \{x : -\infty < x < +\infty\}$
- (c)  $F(x) = \sqrt{4 + x^2}$  where  $Dom(F) = \{x : -\infty < x < +\infty\}$
- (d)  $F(x) = \sqrt{x^2 + x + 1}$

13. [2 marks] Solve the inequality
- $x^2 - 3x + 2 < 0$
- for
- $x$
- .

- (a)  $\{x : -2 < x < -1\}$
- (b)  $\{x : 1 < x < 2\}$
- (c)  $\{x : -\infty < x < 1\}$
- (d)  $\{x : 2 < x < \infty\}$

14. Determine an interval where the graph of the function defined by the polynomial
- $p(x) = x^4 - 6x^3 + 12x^2$
- is concave up.

- (a)  $\{x : -5 < x < 0\}$
- (b)  $\{x : -\infty < x < 1.78\}$
- (c)  $\{x : 2 < x < \infty\}$
- (d)  $\{x : 1 < x < 2\}$

15. [2 marks] Which of the following functions has a point of inflection at  $x = 0$ ?

(a)  $f(x) = x^2 + x + 1$

(b)  $f(x) = -x^2 - 4$

(c)  $f(x) = 2x^3 + 6$

(d)  $f(x) = x^4 + 12$

16. [2 marks] For what values of  $x$  is the function  $f(x) = \frac{1}{x^2 - 1}$  increasing?

(a)  $x > 0$

(b)  $x < 0$

(c)  $0 < x < 1$

(d)  $x > 1$

17. [2 marks] Find all the critical points of  $f(x) = x^3 - 3x + 2$

(a)  $x = -1, x = 1$

(b)  $x = 0, x = 1$

(c)  $x = -2, x = 0$

(d)  $x = 0, x = 1, x = 2$

18. [2 marks] An antiderivative of  $f(x) = \cos(3x + 6)$  is given by

(a)  $\sin(3x + 6)$

(b)  $-3 \sin(3x + 6)$

(c)  $\frac{\sin(3x + 6)}{3}$

(d)  $\sin(3x^2 + 6x)$

19. [2 marks] Evaluate  $\int x^2 3^{2x^3+1} dx$

(a)  $\frac{\ln 3 3^{2x^3+1}}{6}$

(b)  $6x^2 3^{2x^3+1}$

(c)  $3^{2x^3+1}$

(d)  $\frac{3^{2x^3+1}}{6 \ln 3}$

20. [2 marks] Evaluate  $\int_0^4 x \sqrt{2x+1} dx$

(a) 9

(b)  $\frac{298}{15}$

(c)  $\frac{3}{4}$

(d)  $\frac{1}{2}$

21. [2 marks] The value of  $\int \frac{dx}{x \ln x}$  is

(a)  $\ln(\ln x) + C$

(b)  $\ln \ln |x| + C$

(c)  $\ln |x| + C$

(d)  $\ln |\ln x| + C$

22. [2 marks] The most general antiderivative of  $x^2 e^{3x}$  is given by

(a)  $\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27} e^{3x} + C$

(b)  $\frac{1}{9}x^3 e^{3x} + C$

(c)  $\frac{1}{9}x^2 e^{3x} + C$

(d)  $\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + C$

23. [2 marks] Evaluate and simplify the indefinite integral:  $\int e^{2x} \sin 3x \, dx$ .

(a)  $-\frac{9}{13} e^{2x} \cos 3x - \frac{2}{13} e^{2x} \sin 3x + C$

(b)  $\frac{1}{13} e^{2x} \cos 3x + \frac{1}{13} e^{2x} \sin 3x + C$

(c)  $\frac{2}{13} e^{3x} \cos 2x + \frac{9}{13} e^{3x} \sin 2x + C$

(d)  $-\frac{3}{13} e^{2x} \cos 3x + \frac{2}{13} e^{2x} \sin 3x + C$

24. [2 marks] Evaluate the improper integral  $\int_0^{\infty} x^3 e^{-x} \, dx$ .

(a) 6

(b) -8

(c) 0

(d) 1

25. [2 marks] The form of the partial fraction decomposition of the rational function

$$\frac{x-1}{(x^2+1)(x^2-2x-3)}$$

is

(a)  $\frac{A}{x^2+1} + \frac{C}{x-3}$

(b)  $\frac{Ax+B}{x^2+1} + \frac{C}{x-3} + \frac{D}{x+1}$

(c)  $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-3}$

(d)  $\frac{Ax+B}{x^2+1} + \frac{C}{x+3} + \frac{D}{x+1}$

where  $A, B, C, D$  are constants to be determined.

26. [2 marks] Evaluate the indefinite trigonometric integral

$$\int \sin^3 x \cos^2 x \, dx.$$

- (a)  $\frac{\sin^4 x \cos^2 x}{4} + C$   
(b)  $\frac{\sin^3 x \cos^3 x}{3} + C$   
(c)  $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$   
(d)  $\frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + C$

where  $A, B, C, D$  are constants to be determined.

27. [2 marks] The area enclosed by the intersection of the two curves defined by  $y = 1 - x$  and  $y = 2x^2$  is given by which of the following definite integrals?

- (a)  $\int_{-1}^{\frac{1}{2}} (2x^2 - 1) \, dx$   
(b)  $\int_{-1}^{\frac{1}{2}} (1 - x - 2x^2) \, dx$   
(c)  $\int_{-1}^{\frac{1}{2}} (2x^2 - x + 1) \, dx$   
(d)  $\int_{-1}^2 (x - 2x^2) \, dx$

28. [2 marks] Which of the following expressions gives the **volume** of the solid of revolution obtained when the region bounded by the graphs of  $y = 2x$  and  $y = 4x^2$  is revolved about the  **$y$ -axis**?

- (a)  $I = \int_0^{\frac{1}{2}} 2\pi(2x - 4x^2) \, dx$   
(b)  $I = \int_0^{\frac{1}{2}} 2\pi x (4x^2 - 16x^4) \, dx$   
(c)  $I = \int_0^{\frac{1}{2}} 2\pi x (2x - 4x^2) \, dx$   
(d)  $I = \int_0^1 2\pi y (\sqrt{y} - y) \, dy$

29. [2 marks] The center of mass of a thin quarter circle of radius  $R$  having uniform density and situated in the first quadrant is given by which of the following points?

(a)  $\left(\frac{4\pi}{R}, \frac{4\pi}{R}\right)$

(b)  $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$

(c)  $\left(\frac{\pi}{R}, \frac{\pi}{R}\right)$

(d)  $\left(\frac{R}{\pi}, \frac{R}{\pi}\right)$

30. [2 marks] The general solution of the differential equation

$$y^2 \frac{dy}{dt} = t^2 e^{-y^3} \ln t$$

is given by,

(a)  $\frac{1}{3}e^{y^3} = \frac{1}{3}t^3 \ln t - \frac{t^3}{9} + C$

(b)  $e^{y^3} = \frac{1}{2}t^2 \ln 2t + \frac{1}{9}e^t + C$

(c)  $\frac{1}{3}e^{y^3} = \frac{1}{2}t^2 \ln t - t + C$

(d)  $e^{y^2} = \frac{1}{3}t^3 \ln t - \frac{t^3}{9} + C$

**Extra Pages for Rough Work: DO NOT UNSTAPLE**



**Extra Pages for Rough Work: DO NOT UNSTAPLE**

**Extra Pages for Rough Work: DO NOT UNSTAPLE**

**Extra Pages for Rough Work: DO NOT UNSTAPLE**