Exercises: Differential Equations

Solve the given first order linear equations and/or initial value problems. If no initial value is given find its general solution.

1. $y'(x) + 2y(x) = 4x$.

   Answer: $Ce^{-2x} + 2x - 1$, where C is a constant.

2. $y' - (\tan x)y = \sec x$, $\quad y(0) = 0$.

   Answer: $y(x) = x \sec x$.

3. $y'(x) + 2xy(x) = xe^{-x^2}$.

   Answer: $e^{-x^2} \left(C + \dfrac{x^2}{2} \right)$ where C is a constant.

4. $y' + y = \cos x$.

   Answer: $Ce^{-x} + \dfrac{1}{2} (\cos x + \sin x)$ where C is a constant.

5. $y' + ay = e^{mx}$, where a, m are constants.

   Answer:
   $$
   y(x) = \begin{cases} (C + x)e^{mx}, & \text{if} \quad m = -a, \\[2mm] Ce^{-x} + \dfrac{e^{mx}}{m + a}, & \text{if} \quad m \neq -a. \end{cases}
   $$

6. $xy' + y = e^x$, $\quad y(a) = b$, $(a \neq 0)$ are given constants.

   Answer: $y(x) = \dfrac{e^x + ab - e^a}{x}$.

7. $y' = \dfrac{y + 1}{x}$ $\quad y(1) = 0$.

   Answer: $y(x) = x - 1$.

8. $y' - 3y = e^{2x}$, $\quad y(0) = 3$.

   Answer: $y(x) = 4e^{3x} - e^{2x}$.

9. $y' + e^x y = e^{2x}$,

   Answer: $y(x) = Ce^{-e^x} + e^x - 1$, where C is a constant.

10. $y' = ax + by + c$, where a, b, c are constants.

    Answer: $a + bc + abx + b^2 y = Ce^{bx}$, where C is a constant.