Chapter 7

Solutions

7.1 Exercise Set 34

1. $-5x + C$. Use Table 7.2 with $r = 1$, $\square = x$, $c = -5$.
2. $x + C$.
3. C. Use Table 7.2 with $r = 1$, $\square = 0$.
4. $\frac{1}{1.6}\, x^{1.6} + C$. Use Table 7.2 with $r = 0.6$, $\square = x$.
5. $\frac{3}{2}x^2 + C$.
6. $\frac{1}{2}x^2 - x + C$.
7. $\frac{1}{3}x^3 + x + C$. See Example 286.
8. $\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$.
9. $\frac{3}{2}x^2 + C$. (Actually, this is the same as Exercise 5 above.)
10. $x^4 + x^2 - 1.314\, x + C$. See Example 286.
11. $\frac{1}{3}(2x-2)^{3/2} + C$.
12. $\frac{2}{9}(3x+4)^{3/2} + C$.
13. $-\frac{2}{3}(1-x)^{3/2} + C$. See Example 287.
14. $\frac{1}{12}(4x^2+1)^{3/2} + C$.
15. $-\frac{1}{6}(1-2x^2)^{3/2} + C$.
16. $\frac{1}{3.5}(1+x^2)^{1.75} + C$.
17. $\frac{1}{5}(2+x^3)^{5/3} + C$.
18. $-\frac{1}{54}(4+9x^4)^{3/2} + C$. See Example 291.
19. $\frac{1}{3.6}(1+x^{2.4})^{3/2} + C$.
20. $\mathcal{F}(x) = \frac{1}{4}\sin^4 x - 1$.
21. $\mathcal{F}(x) = \frac{1}{3}(1 - \cos^3 x)$.
22. $\mathcal{F}(x) = \frac{1}{2}(e^{-2} - e^{-2x})$.
23. $\dfrac{y^4(x)}{4} = \dfrac{x^3}{3} + \dfrac{1}{4}$. See Example 292.
24. $y = x^4 - 1$.
25. $y(x) = x^4 - 1$. See Example 295.

7.2 Exercise Set 35

1. $\frac{3}{2}$. $\displaystyle\int_0^1 3x\ dx = \left.\frac{3x^2}{2}\right|_0^1 = \frac{3(1^2)}{2} - 0 = \frac{3}{2}$.

2. $-\frac{1}{2}$. $\displaystyle\int_{-1}^0 x\ dx = \left.\frac{x^2}{2}\right|_{-1}^0 = -\frac{1}{2}$.

3. 0. $\displaystyle\int_{-1}^1 x^3\ dx = \left.\frac{x^4}{4}\right|_{-1}^0 = 0$; (note: x^3 is an odd function.)

4. $-\frac{4}{3}$. $\displaystyle\int_0^2 (x^2 - 2x)\ dx = \left.\frac{x^3}{3} - x^2\right|_0^2 = (\frac{2^3}{3} - 2^2) - 0 = -\frac{4}{3}$.

5. 16. $\displaystyle\int_{-2}^2 (4 - 4x^3)dx = 4x - x^4\Big|_{-2}^2 = 16$.

6. $\frac{1}{2}$. $\displaystyle\int_0^{\pi/2} \sin x\ \cos x\ dx = \int_0^{\pi/2} \sin x \left(\frac{d}{dx}\sin x\right)\ dx = \left.\frac{\sin^2 x}{2}\right|_0^{\pi/2} = \frac{1}{2}$.

7. $\frac{2}{3}$. Let $\square = \cos x$. Then $D\square = -\sin x$. So, $\mathcal{F}(x) = -\frac{1}{3}\cos^3 x + C$ and, by definition, $\displaystyle\int_0^{\pi} \cos^2 x \sin x\ dx = -\left.\frac{\cos^3 x}{3}\right|_0^{\pi} = \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3}$.

8. $\frac{1}{4}$. $\displaystyle\int_{-\pi}^{\pi/2} \sin^3 x\ \cos x\ dx = \left.\frac{\sin^4 x}{4}\right|_{-\pi}^{\pi/2} = \frac{1}{4}$.

9. -0.26. (Notice that the upper limit 1.2 of the integral is less than the lower one, namely 1.5; nevertheless we can proceed in the usual way.) $\displaystyle\int_{1.5}^{1.2} (2x - x^2)\ dx = \left.x^2 - \frac{x^3}{3}\right|_{1.5}^{1.2} = -0.26$.

10. $\frac{\pi}{2}$. $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\ dx = \text{Arcsin}\ x\Big|_0^1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$.

11. $\frac{1}{2}(e-1)$. $\displaystyle\int_0^1 xe^{x^2}dx = \int_0^1 e^{x^2}\frac{d}{dx}\left(\frac{x^2}{2}\right)\ dx = \left.\frac{1}{2}e^{x^2}\right|_0^1 = \frac{1}{2}(e-1)$.

12. $2(1 - e^{-4})$.

$$\begin{aligned}\int_0^2 4xe^{-x^2}dx &= \int_0^4 2e^{-x^2}\left(\frac{d}{dx}x^2\right)\ dx \\ &= -2e^{-x^2}\Big|_0^2 = 2(1-e^{-4}).\end{aligned}$$

13. $\frac{2}{\ln 3}$. If we set $f(x) = 3^x$ then $f'(x) = 3^x\ \ln 3$. So $\displaystyle\int 3^x\ dx = \frac{3^x}{\ln 3} + C$. Thus $\displaystyle\int_0^1 3^x\ dx = \left.\frac{3^x}{\ln 3}\right|_0^1 = \frac{3}{\ln 3} - \frac{1}{\ln 3} = \frac{2}{\ln 3}$.

14. $\frac{1}{3}\left(e^{3\triangle} - e^{3\square}\right)$.

15. 0.1340. $\displaystyle\int_0^{0.5} \frac{x}{\sqrt{1-x^2}}\ dx = -(1-x^2)^{1/2}\Big|_0^{0.5} = 1 - \sqrt{0.75} \approx 0.1340$.

16. $\frac{1}{\ln 2}$. We know that $D(a^{\square}) = a^{\square}\ D(\square)\ln a$, where D as usual denotes the operator of taking derivative. It follows $\displaystyle\int a^{\square}\ \frac{d\square}{dx}\ dx = \frac{a^{\square}}{\ln a} + C$. Now, setting $a = 2$, $\square = x^2 + 1$, and $D\square = 2x$, we see that

$$\int_0^1 x\ 2^{x^2+1}\ dx = \frac{1}{2}\left.\frac{2^{x^2+1}}{\ln 2}\right|_0^1 = \frac{2}{\ln 2} - \frac{1}{\ln 2} = \frac{1}{\ln 2}.$$

17. $\dfrac{\sqrt{2}-1}{2}$.

$$\begin{aligned} I &\equiv \int_0^{\sqrt{\pi}/2} x\sec(x^2)\tan(x^2)\ dx \\ &= \int_0^{\sqrt{\pi}/2} \frac{1}{2}\frac{d}{dx}\sec(x^2)\ dx \\ &= \frac{1}{2}\sec(x^2)\Big|_0^{\sqrt{\pi}/2} = \frac{1}{2}\left(\sec\frac{\pi}{4} - \sec 0\right) = \frac{1}{2}(\sqrt{2}-1). \end{aligned}$$

18. 0. Let $\square = x^2$ So $D\square = 2x$ and the antiderivative looks like

$$\frac{1}{2}\int \frac{1}{1+\square^2}\frac{d\square}{dx}\ dx,$$

which reminds one of the derivative of the Arctangent function. In fact,

$$\int_{-1}^{1} \frac{x}{1+x^4}\ dx = \frac{1}{2}\tan^{-1}x^2\Big|_{-1}^{1} = \frac{1}{2}(\tan^{-1}1 - \tan^{-1}1) = 0.$$

(Notice that 0 is the expected answer because the integrand is an odd function.)

19. Following the hint, we have $\dfrac{d}{dx}\displaystyle\int_0^{x^2} e^t\ dt = e^{x^2}\frac{d}{dx}x^2 = 2xe^{x^2}$.

20. These identities can be seen from the respective symmetry in the graph of f. Here is an analytic argument. Assume that f is even: $f(-x) = f(x)$. Let $\mathcal{F}(x) = \int_0^x f(t)dt$, $(-\infty < x < \infty)$. Then $\frac{d}{dx}\mathcal{F}(x) = f(x)$ and

$$\begin{aligned} \int_{-x}^{x} f(t)dt &= \int_{-x}^{0} f(t)dt + \int_0^x f(t)dt \\ &= -\int_0^{-x} f(t)dt + \int_0^x f(t)dt = -\mathcal{F}(-x) + \mathcal{F}(x). \end{aligned}$$

Thus we will have $\int_{-x}^{x} f(t)dt = 2\int_0^x f(t)dt$ if we can show $-\mathcal{F}(-x) = \mathcal{F}(x)$. Let $\mathcal{G}(x) = -\mathcal{F}(-x)$. We are going to show $\mathcal{G} = \mathcal{F}$. Now

$$\begin{aligned} \frac{d}{dx}\mathcal{G}(x) &= \frac{d}{dx}(-\mathcal{F}(-x)) = -\left(\frac{d}{dx}\mathcal{F}(-x)\right) \\ &= -\big(\mathcal{F}'(-x)\cdot(-1)\big) = \mathcal{F}'(-x) = f(-x) = f(x). \end{aligned}$$

Thus, by the Fundamental Theorem of Calculus, $\mathcal{G}(x) = \int_0^x f(t)\ dt + C$ for some constant C, or $\mathcal{G}(x) = \mathcal{F}(x) + C$. Now $\mathcal{G}(0) = -\mathcal{F}(-0) = -\mathcal{F}(0) = -0 = 0$, which is the same as $\mathcal{F}(0)$ $(= 0)$. So C must be zero. Thus $\mathcal{G} = \mathcal{F}$. Done! (The second part of the exercise which involves an even function f can be dealt with in the same manner.)

7.3 Exercise Set 36

1. $\displaystyle\sum_{i=1}^{10} i$.

2. $\displaystyle\sum_{i=1}^{9} (-1)^{i-1}$, or $\displaystyle\sum_{i=1}^{9} (-1)^{i+1}$, or $\displaystyle\sum_{i=0}^{8} (-1)^{i}$.

3. $\displaystyle\sum_{i=1}^{5} \sin i\pi$.