STAT 2509 A
Assignment #5

SOLUTION // 48

1. [31 marks]

[14.5] (a)

[1/2] $$TSS = \sum_{i=1}^{3}\sum_{j=1}^{8} y_{ij}^2 - \frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{8} y_{ij}\right)^2}{bk} = 1006 - \frac{(150)^2}{24}$$ = 1 006 – 937.5 = 68.5 [1/2]

[1/2] $$SST_r = \sum_{i=1}^{3}\frac{T_i^2}{b} - \frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{8} y_{ij}\right)^2}{bk} = \left[\frac{(50)^2}{8} + \frac{(60)^2}{8} + \frac{(40)^2}{8}\right] - \frac{(150)^2}{24} =$$ 962.5 – 937.5 = 25 [1/2]

[1/2] $$SSB = \sum_{j=1}^{8}\frac{B_j^2}{k} - \frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{8} y_{ij}\right)^2}{bk} =$$

$$= \left[\frac{(14)^2}{3} + \frac{(22)^2}{3} + \frac{(18)^2}{3} + \frac{(19)^2}{3} + \frac{(23)^2}{3} + \frac{(17)^2}{3} + \frac{(18)^2}{3} + \frac{(19)^2}{3}\right] - \frac{(150)^2}{24} =$$ 956 – 937.5 = 18.5[1/2]

[1/2] $SSE = TSS - SST_r - SSB =$ 25 [1/2]

[1/2] $MST_r = \frac{SST_r}{k-1} = \frac{25}{2}$ = 12.5 [1/2]

[1/2] $MSB = \frac{SSB}{b-1} = \frac{18.5}{7}$ = 2.642857143 [1/2]

[1/2] $MSE = \frac{SSE}{(b-1)(k-1)} = \frac{25}{14} =$ 1.785714286 [1/2]

[1/2] $F_T = \frac{MST_r}{MSE} =$ 6.999999999 [1/2], [1/2] $F_B = \frac{MSB}{MSE} =$ 1.48 [1/2]

Source	*d.f.*	*SS*	MS	*F*
Treatments	2	25	12.5	6.999999999
Blocks	7	18.5	2.642857143	1.48
Error	14	25	1.785714286	
Total	23	68.5		
	[1/2]	[1/2]	[1/2]	[1/2]

(1/2 mark for each column, if values are entered correctly)

$H_0 : \mu_I = \mu_{II} = \mu_{III}$; **[1]** $\alpha = 0.01$

H_a : *at least one of the* $\mu' s \neq$

test-statistics: $F_T = \dfrac{MST_r}{MSE} = \underline{6.999}$

R.R: we reject H_0 if $F_T > F_{\alpha(k-1,(b-1)(k-1))} = F_{0.01(2,14)} = 6.51$ **[1]**

Since F_T = 6.999 > 6.51 **[1/2]**, we reject H_0 **[1/2]** and conclude that at 1% level of significance there is an evidence to indicate that mean toxic effects of the 3 chemicals differ. **[1/2]**

[3.5] (b)

$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8$; **[1]** $\alpha = 0.01$

H_a : *at least one of the* $\beta' s \neq$

test-statistics: $F_B = \dfrac{MSB}{MSE} = \underline{1.48}$

R.R: we reject H_0 if $F_B > F_{\alpha(b-1,(b-1)(k-1))} = F_{0.01(7,14)} = 4.28$ **[1]**

Since F_B = 1.48 $\not>$ 4.28 **[1/2]**, we do not reject H_0 **[1/2]** and conclude that at 1% level of significance there is not enough evidence to indicate that there are differences between rats (or differences between blocks). **[1/2]**

[2] (c) **Which chemicals differ? Tukey's h.s.d.**

1) Calculate $\binom{k}{2} = \binom{3}{2} = 3$ pairs of $|\bar{y}_i - \bar{y}_j|$ for $H_0 : \mu_i = \mu_j$ vs $H_a : \mu_i \neq \mu_j$,

for i, j = 1, 2, 3
$i \neq j$

h.s.d. = **[1/2]** $q_\alpha(k,(b-1)(k-1))\sqrt{\dfrac{MSE}{b}} = q_{0.01}(3,14)\sqrt{\dfrac{1.785714286}{8}} =$

$= (4.89)(0.472455591) = \underline{2.310307}$ **[1/2]**

[1/2]

$$\bar{y}_I = \frac{T_1}{b} = \frac{50}{8} = 6.25, \qquad \bar{y}_{II} = \frac{T_2}{b} = \frac{60}{8} = 7.5, \qquad \bar{y}_{III} = \frac{T_3}{b} = \frac{40}{8} = 5$$

2) $|\bar{y}_I - \bar{y}_{II}| = 1.25 \not> 2.310307 \Rightarrow \mu_I = \mu_{II}$

$|\bar{y}_I - \bar{y}_{III}| = 1.25 \not> 2.310307 \Rightarrow \mu_I = \mu_{III}$

$|\bar{y}_{II} - \bar{y}_{III}| = 2.5 > 2.310307 \Rightarrow \underline{\underline{\mu_{II} \neq \mu_{III}}}$ **[1/2]**

i.e. there are differences between chemicals (II & III).

[8] (d) **Non-parametric Analysis (Friedman-Rank test)**

Assume: 1) R.B.D. (given) **[1/2]**
2) in each chemical-rat combination we have populations with approximately the same shape **[1/2]** and same spread **[1/2]**
3) no interactions **[1/2]** between chemicals and rats

First we need to rank the observations from smallest to the largest within each block:

Rat Number

Chemical	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
I	6 (3)	9 (2.5)	6 (2)	5 (1)	7 (1)	5 (1.5)	6 (2)	6 (1.5)	$T_{R_1} = 14.5$ **[1/2]**
II	5 (2)	9 (2.5)	9 (3)	8 (3)	8 (2.5)	7 (3)	7 (3)	7 (3)	$T_{R_2} = 22$ **[1/2]**
III	3 (1)	4 (1)	3 (1)	6 (2)	8 (2.5)	5 (1.5)	5 (1)	6 (1.5)	$T_{R_3} = 11.5$ **[1/2]**

(***Check:*** $\frac{bk(k+1)}{2} = \frac{24(4)}{2} = 48$

$\sum_{i=1}^{3} T_{R_i} = 14.5 + 22 + 11.5 = 48$) optional

$H_0: Md_1 = Md_2 = Md_3$; **[1]** $\alpha = 0.01$

H_a : *at least one of the* $Md's \neq$

test-statistics:
[1/2]

$$F_R = \frac{12}{bk(k+1)}\left[\sum_{i=1}^{3} T_{R_i}^2\right] - 3b(k+1) = \frac{12}{24(4)}\left[(14.5)^2 + (22)^2 + (11.5)^2\right] - 3(8)(4) =$$

= 0.125(826.5) – 96 = 103.3125 – 96 = <u>7.3125</u> **[1/2]**

R.R: we reject H_0 if $F_R > \chi^2_{\alpha;(k-1)} = \chi^2_{0.01;(2)}$ = 9.210 **[1]**

Since F_R = 7.3125 $\not>$ 9.210 **[1/2]**, we do not reject H_0 **[1/2]** and conclude that at 1% level of significance there is not enough evidence to say that the medians of 3 different chemicals differ. (i.e. there are no differences between treatments) **[1/2]**

No need for follow-up analysis.

[3] (e) **[1]**

Tests of Between-Subjects Effects

Dependent Variable: toxicity

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	43.500[a]	9	4.833	2.707	.046
Intercept	937.500	1	937.500	525.000	<.001
chemical	25.000	2	12.500	7.000	.008
rat	18.500	7	2.643	1.480	.252
Error	25.000	14	1.786		
Total	1006.000	24			
Corrected Total	68.500	23			

a. R Squared = .635 (Adjusted R Squared = .400)

Post Hoc Tests **[1]**

Multiple Comparisons

Dependent Variable: toxicity

Tukey HSD

(I) chemical	(J) chemical	Mean Difference (I-J)	Std. Error	Sig.	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Chemical I	Chemical II	-1.25	.668	.184	-3.56	1.06
	Chemical III	1.25	.668	.184	-1.06	3.56
Chemical II	Chemical I	1.25	.668	.184	-1.06	3.56
	Chemical III	2.50*	.668	.006	.19	4.81
Chemical III	Chemical I	-1.25	.668	.184	-3.56	1.06
	Chemical II	-2.50*	.668	.006	-4.81	-.19

$\mu_{II} \neq \mu_3$

Based on observed means.

The error term is Mean Square(Error) = 1.786.

*. The mean difference is significant at the .01 level.

or (either the plot on top or on the next page)

toxicity

Tukey B[a,b]

chemical	N	Subset 1	Subset 2
Chemical III	8	5.00	
Chemical I	8	6.25	6.25
Chemical II	8		7.50

Means for groups in homogeneous subsets are displayed.
Based on observed means.
The error term is Mean Square(Error) = 1.786.

a. Uses Harmonic Mean Sample Size = 8.000.

b. Alpha = .01.

$\mu_{II} \neq \mu_{III}$

Friedman Test

Ranks

	Mean Rank
chemical1	1.81
chemical2	2.75
chemical3	1.44

$T_{R_1} = 1.81 \times 8 = 14.48 \doteq 14.5$

$T_{R_2} = 2.75 \times 8 \simeq 22$

$T_{R_3} = 1.44 \times 8 = 11.52 \doteq 11.5$

[1]

Test Statistics[a]

N	8
Chi-Square	8.357
df	2
Asymp. Sig.	.015

a. Friedman Test

χ^2 approx. to $F_R \doteq 7.3125$

2. [8 marks]

a =2, b = 3, r = 2 and ab = 6, Factor A = Foreman, Factor B = House Design

$A_1 = R_1 = \sum y_{1jk}$ = 82.8, $A_2 = R_2 = \sum y_{2jk}$ = 70.4

$B_1 = C_1 = \sum y_{i1k}$ = 41.8, $B_2 = C_2 = \sum y_{i2k}$ = 47.5, $B_3 = C_3 = \sum y_{i3k}$ = 63.9

$(AB)_{11} = \sum y_{11k}$ = 21.3, $(AB)_{12} = \sum y_{12k}$ = 23.9, $(AB)_{13} = \sum y_{13k}$ = 37.6,

$(AB)_{21} = \sum y_{21k}$ = 20.5, $(AB)_{22} = \sum y_{22k}$ = 23.6, $(AB)_{23} = \sum y_{23k}$ = 26.3

$G.T. = R_1 + R_2$ = 82.8+70.4 = 153.2 (= $C_1 + C_2 + C_3$)

[5] (a)

ANVOA **[1]**

Tests of Between-Subjects Effects
Dependent Variable: Profits

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	97.927[a]	5	19.585	50.219	<.001
Intercept	1955.853	1	1955.853	5015.009	<.001
Foreman	12.813	1	12.813	32.855	.001
HouseDesign	65.822	2	32.911	84.387	<.001
Foreman * HouseDesign	19.292	2	9.646	24.733	.001
Error	2.340	6	.390		
Total	2056.120	12			
Corrected Total	100.267	11			

a. R Squared = .977 (Adjusted R Squared = .957)

$H_0: (\alpha\beta)_{ij} = 0 \ \ \forall ij$ (or Foreman and House Design do not interact), $\alpha = 0.05$
$H_a:$ at least one $(\alpha\beta)_{ij} \neq 0$ (they interact) **[1]**

test-statistic: $F_{AB} = \dfrac{MS(AB)}{MSE} = 24.733$ **[1/2]** (or p-value = 0.001 **[1/2]**)

R.R. we reject H_0 if $F_{AB} > F_{\alpha;((a-1)(b-1),ab(r-1))} = F_{0.05;(2,6)} = 5.14$ **[1]**
(or if p-value < $\alpha = 0.05$)

-Since F_{AB} = 24.733 > 5.14 **[1/2]** (or since p-value = 0.001 < 0.05), we reject H_0 **[1/2]** and conclude that at 5% level of sign., we have an evidence that there is an interaction between Foreman and the House Design. **[1/2]**

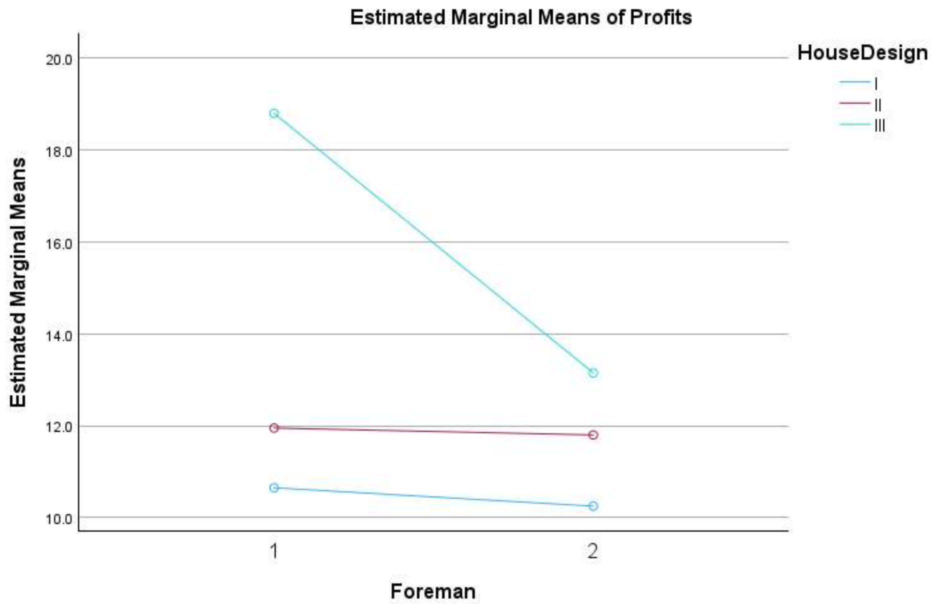
[1] (b)

[1/2] No, since we have significant interaction effect **[1/2]**, we do not proceed with the tests for the main effect.

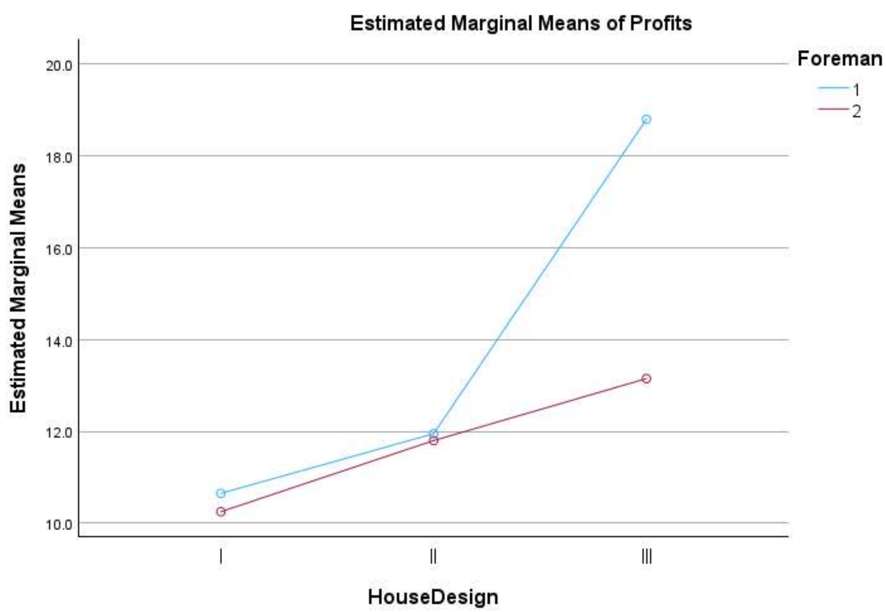
[2] (c)

Interaction plot (see below) suggests that there is a (mild) interaction present**[1/2]**, which confirms **[1/2]** the results in part a).

[1]



or (either one of the plots is fine)



3. [9 marks]

H_0: $p_1 = p_2 = p_3 = \frac{1}{3}$,
H_a: at least one p_i is different } **[1]**; $\alpha = 0.05$

Assumptions: 1) a random sample **[1/2]** taken from
2) populations distributed with Multinomial distribution **[1/2]**
3) n is large enough for χ^2 to apply **[1/2]**

$E_1 = E_2 = E_3 = np_i = 300(\frac{1}{3}) = 100$ **[1.5] ….** 1/2 mark for each E_i

test-statistics: **[1/2]** $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} =$ **[1/2]** $\frac{(85-100)^2}{100} + \frac{(93-100)^2}{100} + \frac{(122-100)^2}{100} = \underline{\underline{7.58}}$ **[1/2]**

R.R.: we reject H_0 if $\chi^2 > \chi^2_{\alpha;(k-1)} = \chi^2_{0.05,(2)} = 5.99147$ **[1]**

Since 7.58 > 5.99147**[1/2]**, we reject H_0 **[1/2]** and conclude that at $\alpha = 0.05$, there is enough evidence to say that the three coffee types don't sell equally. **[1/2]**

Coffee_Type **[1/2]**

	Observed N	Expected N	Residual
Latte	85	100.0	-15.0
Espresso	93	100.0	-7.0
Cappuccino	122	100.0	22.0
Total	300		

Test Statistics **[1/2]**

	Coffee_Type
Chi-Square	7.580[a]
df	2
Asymp. Sig.	.023

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 100.0.