STAT 2509 A

Assignment #1

(Review of STAT 2507)

SOLUTION // 35

1. [1 mark]

a) (ii) a parameter **[1/2]**
b) (iii) a statistic **[1/2]**

2. [0.5 mark]

b) mean & median **[1/2]**

3. [0.5 mark]

(iii) statistical inference **[1/2]**

4. [5 marks]

a)

(i) parameter: is a descriptive measure of a population **[1/2]**. It is a fixed constant **[1/2]**

(ii) statistic: is a descriptive measure of a sample **[1/2]**. It varies from sample to sample **[1/2]**

b) (i) σ^2 - parameter **[1/2]** (ii) μ - parameter **[1/2]** (iii) $\hat{\beta}_1$ - statistic **[1/2]**
(iv) s^2 - statistic **[1/2]** (v) β_0 - parameter **[1/2]** (vi) $\bar{x}$ - statistic **[1/2]**

5. [7 marks]

a) Mercury concentration in a sample of tuna **quantitative[1/2] & continuous[1/2]**
b) Fast-food establishment preferred by a student (McDonald, Burger King, A&W) **purely categorical (or qualitative) [1]**
c) Score (0 – 100) on a placement examination **quantitative[1/2] & continuous[1/2]**
d) Taste ranking (excellent, good, fair, poor) **categorical (or qualitative) & ranked [1]**
e) Colour of rose bush **purely categorical (or qualitative) [1]**
f) The number of defective lightbulbs in a package of 4 bulbs **quantitative [1/2] & discrete [1/2]**
g) Dress size: 3, 5, 7, 9, 11, 13, 15, 17 **categorical (or qualitative) & ranked [1]**

6. [6 marks]

a) Two-sided hypotheses: a hypothesis testing problem of the form:

$$H_0 : \mu = \mu_0 \ \textbf{[1/2]} \quad \text{versus} \quad H_a : \mu \neq \mu_0 \ \textbf{[1/2]}$$

(where μ_0 is a specified (or, given) value).

(Alternatively, you can answer: $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$)

One-sided hypotheses: a hypothesis testing problem of the form:

$H_0 : \mu \leq \mu_0$ versus $H_a : \mu > \mu_0$

Or **[1/2]** **[1/2]**

$H_0 : \mu \geq \mu_0$ versus $H_a : \mu < \mu_0$

(where μ_0 is a specified (or, given) value).

(Alternatively, you can answer: $H_0 : \mu \leq 0$ versus $H_a : \mu > 0$

or $H_0 : \mu \geq 0$ versus $H_a : \mu < 0$

Steps involved:

(1) **[1/2]** State H_0 and H_a.
(2) **[1/2]** Find the value of the test statistic for the hypothesis testing problem.
(3) **[1/2]** Find the rejection (or, critical) region or p-value (if p-value method is used)
(4) **[1/2]** Draw conclusion.

b) **[1]** Type I error: the error that we make when we reject the null hypothesis H_0 when H_0 is actually true and should not be rejected.

(Note (optional): The probability of making Type I error is usually denoted as α.)

[1] Type II error: the error that we make when we do not reject the null hypothesis H_0 when H_0 is actually false and should be rejected.

(Note (optional): the probability of making Type II error is usually denoted as β.)

7. [6 marks] here 0.5 mark for the correct "sign" and 0.5 mark for the correct "value"

a) $z_{0.025}$ = **1.96** **[1]**

b) $z_{0.975}$ = $-$ $z_{0.025}$ = **$-$ 1.96** **[1]**

c) $z_{0.05}$ = **1.645** [1]

d) $t_{0.10;4}$ = **1.533** [1]

e) $-t_{0.10;4}$ = **– 1.533** [1]

f) $t_{0.90;4}$ = $-t_{0.10;4}$ = **– 1.533** [1]

8. [4 marks]

a) (i) E(k)= ***k*** [1/2], (ii) E(kX)= ***kE(X)*** [1/2], (iii) E($X \pm Y$)= ***E(X)*** $\pm$ ***E(Y)*** [1/2]

b) (i) V(k)= ***0*** [1/2], (ii) V(kX)= ***k²V(X)*** [1/2], (iii) V($X \pm Y$)= ***V(X) + V(Y)*** $\pm$ ***2Cov(X,Y)*** [1]

Also show what happens when X and Y are independent of each other?

When X and Y are independent, then they are not related and so Cov(X,Y) = 0, i.e. *V(X±Y)= V(X) + V(Y)* [1/2]

9. [5 marks]

a)

(i) 89.68% $\Rightarrow 1-\alpha =$ **0.8968** $\Rightarrow \alpha =$ **0.1032** $\Rightarrow \alpha/2 =$ **0.0516** $\Rightarrow z_{\alpha/2}$ = **1.63** [1]

(ii) 99.20% $\Rightarrow 1-\alpha =$ **0.9920** $\Rightarrow \alpha =$ **0.0080** $\Rightarrow \alpha/2 =$ **0.0040** $\Rightarrow z_{\alpha/2}$ = **2.65** [1]

(iii) 75.40% $\Rightarrow 1-\alpha =$ **0.7540** $\Rightarrow \alpha =$ **0.2460** $\Rightarrow \alpha/2 =$ **0.1230** $\Rightarrow z_{\alpha/2}$ = **1.16** [1]

b)

- **90% C.I. would be narrower [1/2] than 99.20% C.I.**
- **Because 90% C.I. would have shorter span [1/2] (i.e. it covers smaller interval of values) as it has smaller error** $z_{\alpha/2}\,\sigma/\sqrt{n}$ [1/2]
- **The only way to make 99.20% C.I. of the same width as the 90% C.I. is to increase the sample size *n*.** [1/2]