Corrections to Eugene Fiume's text on

An Introduction to Scientific, Symbolic, and Graphical Computation

as found (mostly) by R. M. Neal, September 1996, updated November 1996

These errors are in the 1st printing (blue cover); most are corrected in the 2nd printing (green cover).

Chapter 0

Page 11:

Figure 0.4: The scale on the vertical axis in this figure is incorrect.

Page 13:

The text uses the term "unit circle" to refer to a circle with a diameter of one. Note that "unit circle" is more usually used to refer to a circle of radius one, with centre at the origin. You may have encountered this latter usage when studying trigonometry. (This non-standard use of "unit circle" also occurs on the pages that follow.)

Page 16:

The `local i;` declaration in the Maple procedure `Pn` is unnecessary. It has no effect, but should be deleted since `i` is never used.

The definition of $C_k = \cos\theta_k$ should be $C_k = \cos\omega_k$.

Page 17:

The sentence:

> We "bottom out" with C_2, which is simply $\cos(\pi/4) = \sqrt{2}/2$.

should be

> We "bottom out" with C_2, which is simply $\cos(2\pi/2^2) = \cos(\pi/2) = 0$.

Equation (2) should be $Q_k = 2^k\sqrt{(1-C_k)/2}$.

The Maple procedure `perim` should be as follows:

```
perim := proc (k)
    2^k * sqrt((1-C(k))/2)    # recurrence avoids Pi
end:
```

Page 18:

The Maple procedure `C` should be as follows:

```
C := proc (k)           # works only for k>=2
    option remember;
    if (k=2) then 0
    else sqrt((C(k-1)+1)/2);
    fi
end:
```

Chapter 1

Page 24:

Exercise 1.5: "If $f : \mathsf{R} \to \mathsf{R}$ is defined as $f(x) = \log_2(x)$" is an incorrect definition, since $\log_2(x)$ is not defined for all $x \in \mathsf{R}$. You should instead take the domain of f to be the positive reals.

The statement: "Hoever, the function $y = \max(x, 1)$ is *not* continuous on $[-1, 1]$." is incorrect. This function is continuous everywhere, and is in fact constant on the interval $[-1, 1]$.

Exercises 1.9 and 1.10: In both exercises, the functions given are not defined at $x = 0$, so it doesn't really make sense to ask whether they are continuous at that point. However, the exercises will make sense if you define the value of y at $x = 0$ to be zero.

Page 25:

Starting on this page, the text talks about defining "functions" in an implicit fashion, for example, a circle of radius r is defined by the equation $F(x, y) = 0$ where $F(x, y) = x^2 + y^2 - r^2$. This usage of the word "function" is incorrect, and is contrary to its definition earlier in the text. A circle is not a function, because it does not map a value for x to a single value for y: instead, any x in the open interval $(-r, +r)$ is associated with *two* values for y, one positive and one negative.

You should watch out for this problem throughout Section 1.4 — indeed, the title of this section should be "Representation of Curves and Surfaces", not "Representation of Functions".

Page 30:

Exercise 1.20: "Is $C(\theta)$ surjective on $[0, 2\pi]$?" This question cannot be answered, since there is no indication of what set is considered to be the target of this function.

Page 56:

"Thus the quantity $(\mathtt{ysf} - 1) \in [-1/2, 1/2]$" should be "Thus the quantity $(\mathtt{ysf} - 1) \in [-1, 0]$".

Chapter 2

Page 77:

The definition of the term "knot" given here is not the usual one. In the more common terminology, a knot is a point where the interpolating function switches from one polynomial to another (as talked about later in this chapter). Lagrange interpolation uses only a single polynomial, so the concept of a knot as it is usually defined does not apply.

Exercise 2.5: Note that `x[i]` here corresponds to t_i in the main text, while `y[i]` corresponds to x_i.

Page 80:

Example 2.3: Here also, `x` and `y` correspond to t and x in the main text.

Page 84:

The statement "We see in Figure 2.9 that just as in the linear case, this [piecewise cubic] interpolation does not change the past [when new points are added]" is not quite right. Adding new points does not change the interpolant for much earlier times, but it does change the interpolant for the times just before the new points, as you can see by comparing carefully the functions in Figures 2.8 and 2.9 over the interval $[4, 5]$.

Page 97:

The text says, "As seen in Figure 2.17, only the two central points are interpolated". Of course, you can't see this from the figure (though it is true), since the plot in the figure doesn't extend to the outer points (at 0 and 3).

Page 100:

Exercise 2.16: The reference to Figure 2.19 should be to Figure 2.18.

Exercise 2.18: "Given five points $x_0, x_1, \ldots, x_5$" should be "Given five points $x_0, x_1, \ldots, x_4$".

Page 101:

"We have deliberately expressed it in a form that will ... allow variation in x as well as y." should instead end with "... allow variation in y and z as well as x."

Page 113:

Clarifications to Exercise 2.31:

First, in the equation for a curve segment, $p(t)$, you should view $p(t)$ as a vector of functions, and α_j as a vector containing the coordinates for observation j. Previous such equations (eg, (39)) have applied to a single coordinate (eg, x), with the other coordinates (eg, y and z) being treated separately. Here, it is crucial that one look at all the coordinates at once, since they may be mixed together by the transformation. Note, however, that $b_j(t)$ is *not* a vector — that is, we are using the *same* basis functions for each coordinate.

Second: Contrary to what is said in the exercise, a translation is not a linear transformation. Translating by $\mathbf{x}$ takes $\mathbf{u}$ to $\mathbf{u}+\mathbf{x}$, and $\mathbf{v}$ to $\mathbf{v}+\mathbf{x}$, and $\mathbf{u}+\mathbf{v}$ to $(\mathbf{u}+\mathbf{v})+\mathbf{x}$. If translation were a linear transformation, $\mathbf{u}+\mathbf{v}$ would go to $(\mathbf{u}+\mathbf{x})+(\mathbf{v}+\mathbf{x})$, which is not the same as $(\mathbf{u}+\mathbf{v})+\mathbf{x}$.

Finally, you can prove the result separately for linear transformations and for translations (making the partition of unity assumptions) in either order — the latter result does not depend on the former.

Chapter 3

Page 125:

Figure 3.1: Note that although the Langrange interpolant is plotted for the interval $[2, 6]$, it is based on values of the function at points over a wider range. Also, the points used to contruct the Langrange interpolant on the left are not the same as those used to construct the cubic

spline interpolant on the right, so one can't really compare the two (though the difference in behaviour in the two plots is qualitatively representative of the differences between these two interpolants' behaviour).

Page 146:

In equation (34), $g(t;\, \sigma)$ should be $g(x;\, \sigma)$.

Page 146:

Figure 3.17: Note that the moving average filter in this figure is defined in continuous time, not in discrete time as in equation (30), and is convolved with itself by the continuous-time convolution of equation (27).

Page 149:

Figure 3.19: There's something wrong with this figure (though the basic idea is correct). If the moving average filter with half-width 7 is discrete, the matching Gaussian should have $\sigma = 12.22$; if the moving average filter is continuous, the matching Gaussian should have $\sigma = 11.43$. The relationship between the filter width, the number of times it is iterated, and the standard deviation of the approximating Gaussian is not all that complex (though I won't give it here), but it isn't just a matter of multiplying by two, as you might think from the text!

Page 162–164:

Regarding the discussion of sampling at increasing rates:

The sampling rates in Figures 3.35, 3.36, 3.37, and 3.38 should be $4/8\pi$, $12/8\pi$, $16/8\pi$, and $24/8\pi$, not the figures given in the text (which are bigger in each case by $1/8\pi$). The sampling rate is the limit of the number samples divided by the amount of time, as the amount of time goes to infinity. It is always the reciprocal of the sample spacing.

You should not take seriously the labelling of the approximation in Figure 3.36 as "undersampled", that in Figure 3.37 as "properly sampled", and that in Figure 3.38 as "oversampled". In all these figures, the sampling rate is higher than the Nyquist rate, and is high enough for Catmull-Rom interpolation to produce at least a rough approximation to the sine wave. In none of the figures is the approximation perfect, and in fact one would need an infinite sampling rate to perfectly reconstruct the sine wave with Catmull-Rom interpolation (which is not the optimal method).

In Figure 3.35, "An undersampled approximation to sine using 5 samples", there are actually 9 sample points shown. The second, fourth, sixth, and eighth points should be deleted from the figure.

Page 168:

The text says: "The sampling theorem tells us how many regularly spaced samples of a continuous function we must carry on a slip of paper in our pockets (or on a diskette or for transmission) to be able to reconstruct that function later". This may be misleading. The sampling theorem says what the sampling *rate* must be. To perfectly reconstruct the function, you must sample at that rate *for an infinite time*. As you might expect, an object such as a function on a continuous domain (even one with no frequencies above a maximum) that can vary in an infinite number of ways cannot be perfectly represented by a finite number of samples.

Chapter 4

Page 178:

In the last part of Example 4.4: "Furthermore, $y(x)$ is simply $f(x)$" should be "Furthermore, $y(t)$ is simply $f(t)$". The formula for the arc length that follows should be

$$\int_a^b \sqrt{1+(f'(t))^2}\,dt$$

Page 187:

Figure 4.10: This figure does not, of course, depict 8 trapezoids. By mistake, it is the same as Figure 4.6.

Page 195:

The third equation on this page — "The final form of the approximation is thus" — should be

$$\int_a^b f \;=\; \sum_{i=-1}^{n-2} \Delta x_{i+1} \int_0^1 C_i(t)\,dt$$

The indexing is a bit different from equation (2) because of the way the C_i are defined using the previous and following points.

Page 202:

The term "adaptive", used here to described a numerical integration method that decreases the point spacing as needed to achieve some level of error, is more commonly used to describe a numerical integration method that uses more points in regions of the domain where the function varies a lot than in other regions where is it smoother.

Exercise 4.8: "The points used in iteration i be reused..." should be "The points used in iteration i can be reused...".

Page 209:

Figure 4.20: Note that the two routines shown must be called in different fashions. The **montecarlo** routine must be passed a function, or name of a function, the **mc** routine must be passed an expression with one indeterminant, which is taken to define a function. For example, the following two calls should both produce something close to 2:

```
montecarlo(sin,[0,Pi],1000);
mc(sin(x),[0,Pi],1000);
```

Page 210:

"If instead we were to roll three dice repeatedly and compute the distribution, we would get a parabolic hump as in Figure 4.22." As you can see from the figure, the distribution is not actually a parabola.

Page 213:

Here and on the next page, the text uses the term "indefinite integral" to refer to integrals in which one or both of the integration limits is at infinity. In the usual terminology, given on page 175, integrals with infinite limits (or with limits where the function value is infinite) are called "improper integrals". An "indefinite integral" is one with no bounds of any kind (what is also called an "antiderivative").

Page 214:

Exercise 4.17: The two candidate transformations between u and x are given backwards. They should be

$$\begin{aligned} u &= -\ln x, \quad \text{and} \\ u &= \frac{x}{1-x}. \end{aligned}$$

The integral to try calculating using these changes of variables should be

$$I = \int_0^\infty \exp(-u^2)\, du$$

Chapter 5

Page 232:

The discussion of representation error at the beginning of Section 5.2 is incorrect. The maximum error in the floating-point representation of a real number depends on the magnitude of the number. This is the fundamental difference between floating-point and fixed-point representations. If we look at numbers whose exponent parts are zero, the maximum error with a five decimal digit representation is 0.5×10^{-5}. In general, with d decimal digits, a number whose exponent part is zero will have a representation error of no more than 0.5×10^{-d}.

Page 235:

The statement of Taylor's Theorem should be as follows:

If derivatives of f up to order $n+1$ exist on the interval $[x_0, x_1]$, containing a, then for any x in this interval, there exists a c within the interval (which may depend on x) such that

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \cdots + \frac{f^{(n)}(a)(x-a)^n}{n!} + \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$$

It follows that for any x in $[x_0, x_1]$

$$|r_n(x)| = |f(x) - p_n(x)| \leq C|x-a|^{n+1}$$

where C is a constant given by

$$C = \frac{|f^{(n+1)}(x_{\max})|}{(n+1)!}$$

where $x_{\max}$ is a point in $[x_0, x_1]$ for which $|f^{(n+1)}(x)|$ takes on its maximum value.

Page 236:

Exercise 5.14: "While the residual r_n provides an upper bound on the error...". As defined on page 233, $r_n(x)$ is the actual error, not a bound. It's magnitude can be bounded using Taylor's Theorem, however.

Page 258:

The integrals defining the Fourier cosine and sine transformations at the top of this page should have bounds of $-\infty$ and $+\infty$, not 0 and $+\infty$. Also, the constant in front should probably be $1/\pi$ instead of $2/\pi$, to be consistent with the later example, though this is really just a matter of convention (and, unfortunately, different authors use different conventions in this regard).

Page 263:

The definition of the *continuous convolution* of two functions f and g should be

$$(f * g)(x) \;=\; \int_{-\infty}^{+\infty} f(y)\, g(x-y)\, dy$$

Chapter 6

Page 281:

Exercise 6.3: The equation following "Show that the solution in parameter t of the intersection of L with plane P is" should be

$$t \;=\; \frac{P(P_0)}{P(P_0) - P(P_1)}$$

Page 282:

In the expression for *spc* in slightly nicer notation, a factor of t should follow the second expression in parentheses.

Page 289:

In the algorithm of section 6.3.1,

evaluate $f_i = f(a + i\Delta x)$ **for** $i = 0, 1, \cdots, 1/\Delta x$.

should be

evaluate $f_i = f(a + i\Delta x)$ **for** $i = 0, 1, \cdots, (b-a)/\Delta x$.

Page 290:

The text says, "A mathematically nice (but computationally inefficient) way of saying that two values u and v differ in sign is to say that $uv < 0$." You should never do this. Aside from being inefficient, when u and v are floating-point numbers, it is quite possible that multiplying u by v will result in floating-point underflow, which will cause the product uv to be considered equal to zero. In the procedure `bisect` that follows, this could cause the interval to be narrowed the wrong way, excluding the true root. It is also possible that computing uv could result in floating-point overflow, which could cause the program to crash. Since Maple allows extremely large exponents, these problems will probably not occur with the program as written, but they would become possible if the calls of `evalf` were replace by calls of `evalhf`, in order to use the machine's hardware floating-point operations (which are much faster than Maple's software operations).

To fix this problem with `bisect`, the statement

```
if (fa*fm < 0) then...
```

should be replaced by

```
if (fa<0 and fm>0) or (fa>0 and fm<0) then...
```

This procedure can also go into an infinite loop if passed a value for `tolI` that is greater than zero but small enough that adding this quantity to a floating-point value in the vicinity of the solution has no effect. To fix this, the condition in the while loop should be changed to

```
while abs(m-a)>tolI and abs(m-b)>tolI do...
```