Computer Science CSC260S — Spring 2000
St. George Campus — University of Toronto

Homework Assignment #3
Due: Monday, March 27, 2000
Partners: not allowed for this assignment
Mark: 10%

1. (30 points)

(a) (20 points) Write a MAPLE procedure called `secant` that implement the secant method for finding a zero of a function f in a variable x. The parameters of your procedure are: the function f, its variable x, the two starting points, the tolerance and the maximum number of iterations. The MAPLE procedure for the Newton-Raphson method in the lecture notes should be instructive. You should follow the general pattern of that procedure.

(b) (10 points) Use the MAPLE commands *solve*, *fsolve* and *plot* to try to find a zero of the function

$$f(x) = 3^{3x+1} - 7 * 5^{2x}.$$

Use this information to find initial approximations for a zero of $f(x)$. Use your secant method of Part (a) to find a zero of $f(x)$ to within 10^{-12}.

2. (20 points) This question is to be done by hand calculations.

Consider the integral $I = \int_0^1 \frac{1}{1+x^2} dx$. Approximate I using the following quadrature rules: (a) midpoint; (b) trapezoid; (c) Simpson's; (d) compound trapezoid with 2 panels; (e) compound Simpson's with 2 panels. Show all your calculations.

3. (20 points) In order to compute the integral: $I = \int_a^b f(x)dx$, we can approximate $f(x)$ in $[a, b]$ using an interpolation scheme, and then computing the integral of the interpolant.

(a) (12 points) Write a MAPLE script that:

- computes the linear, cubic and Lagrange interpolation polynomials associated with the function $f(x)$ using n equidistant points in $[a, b]$ ($x_1 = a, x_n = b$);
- computes the integrals in $[a, b]$ of the cubic, linear and Lagrange interpolation polynomials;
- estimates the three absolute and relative errors.

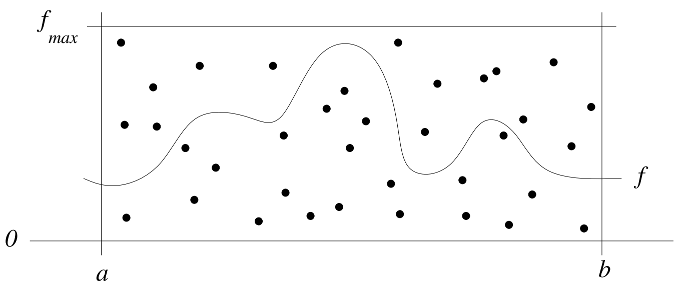
(b) (8 points) Use your script of Part (a) with $n = 11$ to estimate the integral of Question 2, that is,

$$I = \int_0^1 \frac{1}{1+x^2} dx.$$

Provide the absolute and relative errors of your three approximations.

4. (30 points) The "Monte Carlo method" of numerical integration uses random samples to compute the required integral. Section 4.4 in the textbook explains the Monte Carlo approach in detail.

(a) (24 points) Suppose we want to compute the integral of function f in the range $[a, b]$ using the Monte Carlo method. We know that the function takes on values between 0 and f_{max} in that range, where f_{max} is a parameter of the procedure.



The Monte Carlo method generates N random points in the box $[a, b] \times [0, f_{max}]$. It determines what fraction of them lie below the curve of f. Then, the integral of f is approximately that fraction of the box's area.

For example, of the 36 random points shown above, 21 lie below the curve of f. Therefore, f's integral is approximately

$$21/36 * \text{ (area of box)} = 21/36 * (b - a) * f_{max}.$$

Implement the Monte Carlo method in MAPLE. The parameters are a, b, N, and f_{max}.

In MAPLE, the package "stats" provides functions for the generation of pseudorandom numbers. For instance, *stats[random, uniform](10)* provides 10 values pseudorandomly distributed (with a uniform distribution) in 0..1.

Hint: A random point (x, y) lies below the curve of f if $y < f(x)$. Assume that no random point ever lies exactly on the curve.

(b) (6 points) Use your Monte Carlo method of Part (a) with $N = 36$ and a f_{max} value determined by yourself, to estimate the integral

$$I = \int_0^1 \frac{e^x}{1+x} dx.$$