CSC 238F 1999, Assignment 1, St. George Campus, Day Section
Due: Oct. 6, 10:10 AM (beginning of tutorial).

On the cover page of your assignment, you must write **and sign** *the following statement: "I have read and understood the policy on collaboration in homework stated in the webpage of the course." Without such a signed statement your homework will not be marked.*

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1. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined as follows: [10]
   $f(0) = 6$, and
   for all $n \in \mathbb{N}$, $f(n+1) = 5f(n) + 3^n$.

   Use induction to prove that for some positive constant c and for all $n \in \mathbb{N}$, $f(n) \leq c \cdot 5^n$.

   HINT: It may be easier to first prove the stronger statement that $f(n) \leq 7 \cdot 5^n - 3^n$.

   (REMARK: These issues will appear less mysterious when we study recurrences later in the course.)

2. Imagine that we live in a country where the only denominations of stamps are 7 and 8, and so we are interested in what values of postage can be obtained by combining (zero or more) stamps of value 7 with (zero or more) stamps of value 8.

   For this question, you should produce a particular positive integer c and prove the following:

   (a) For every integer $n \geq c$, it is possible to form postage n. [5]

   (b) It is *not* possible to form postage $c - 1$. [5]

3. (In this question, when we say "an n-bit string", we mean a binary string of length n.)

   (a) Let n be a positive integer, and let S be a set of n-bit strings such that no two strings in S differ in *exactly* one position. Prove that S contains no more than 2^{n-1} strings. [10]
   HINT: Use induction.

   (b) **EXTRA CREDIT:** Prove that for every positive integer n, there exists a set S of n-bit strings such that no two strings in S differ in *exactly* one position, and such that S contains exactly 2^{n-1} strings. [5]

4. Define the sequence of integers $a_0, a_1, a_2, \cdots$ as follows: [10]

   $a_0 = 2$,
   $a_1 = 2$,
   $a_2 = 2$,
   $a_i = a_{i-1} + a_{i-2} + a_{i-3}$ for all $i \geq 3$.

   Use complete induction to prove that $a_n < 2^n$ for every integer $n \geq 2$.

5. Prove that the following program halts for every input $x \in \mathbb{N}$. [10]

```
y := x*x         % Precondition: x is a natural number
while (y <> 0) % Do the loop as long as y is not equal to 0
    x := x-1
    y := y-2*x-1
end while
```

Hint: Derive (and prove) a loop invariant whose purpose is to help prove termination. (Note that you are not asked to prove partial correctness here – no postcondition is even specified!)