Abstract

A multibeam satellite system with on-board processing and memory is studied here. In this system, multiple slotted ALOHA up-links carry the traffic from spatially disjoint earth zones to the satellite. Packets are accepted at the satellite if memory is available and are routed to their destination zones. The model considered here will allow more than one transponder to serve a destination zone in each time slot. This is different from the model considered by Chlamtac and Ganz(1986), in which each zone is served by at most a single transponder. When the restriction of Chlamtac and Ganz to conflict-free scheduling is relaxed, the maximum throughput is increased by as much as 40%.
Performance Analysis of a Multibeam Packet Satellite System Using Random Access Techniques

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I Introduction

Chlamtac and Ganz [2] have analyzed the performance of a multibeam satellite switch with on-board processing and memory. Their analysis applies under the condition of conflict-free scheduling. It is of some interest to determine the importance of this condition. Consequently, the present paper drops this condition completely. To get the results, a different method is developed here.

In the system studied here, multiple slotted ALOHA up-links carry the traffic from spatially disjoint earth zones to the satellite. Packets are accepted at the satellite if memory is available and are routed to their destination zones. Packets rejected due to their arriving to a full buffer are retransmitted after a randomized delay. The model considered here will allow more than one transponder to serve a single destination zone at the same time. This is different from the model considered in [2]. If the restriction of Chlamtac and Ganz to conflict-free scheduling is relaxed, the maximum throughput, pictographically illustrated in their paper, is increased by as much as 40%. We formulate this system as a $D^X/D^m/1$ queueing model, which will be explained in the following section. When the buffer size is small, a numerically stable algorithm based on the state-reduction method is used to obtain the equilibrium probabilities of the number of the packets in the system.

When the buffer size is large, we use the model with infinite buffer size to approximate it. The well-known generating function technique will be used to analyze this model. Several other researchers have also used this technique to analyze the model [1]. In order to obtain an explicit formula, we need to separate the boundary probabilities from the non-boundary probabilities in the expression for the generating function. Numerical results indicate that this explicit formula, which is exact for infinite buffers, will give satisfactory approximations for most finite buffers. The only exception is under heavy traffic conditions and with a small buffer size. In this case, the state-reduction method [4, 5] will directly solve the system.
II The Model

The model considered here is the same as that in [2], except that we allow more than one transponder to serve a single zone. Mathematically, consider the imbedded times \( t_k \) \((k = 1, 2, \ldots)\), which are time epochs immediately after the \( k \)th arrival, and define time slot \( k \) as the length of time between time epochs \( t_{k-1} \) and \( t_k \). Let packets arrive in batches of size \( X \), and let the number of packets sent to destinations in a time slot be \( m \), the same as the number of the transponders, if there are at least \( m \) packets at the satellite. This queueing model will be denoted by \( D^X/D^m/1 \). Let \( X_k \) be i.i.d. random variables with the same distribution as \( X \). Then the number of packets \( N_k \) at the satellite at time \( t_k \) \((k = 1, 2, \ldots)\) for the finite buffer case is

\[
N_k = \begin{cases} 
(N_{k-1} - m)^+ + X_k, & \text{if } (N_{k-1} - m)^+ + X_k \leq B \\
B, & \text{otherwise,}
\end{cases} \tag{1}
\]

where \( B \) is the buffer size; and for the infinite buffer case,

\[
N_k = (N_{k-1} - m)^+ + X_k, \tag{2}
\]

where \( X^+ \) is defined by \( X^+ = \max(X, 0) \). Since the number of packets at the satellite at time epoch \( t_k \) only depends on the number of packets at the satellite at the previous time epoch \( t_{k-1} \), the number of packets transmitted, and the number of new arrivals during time slot \( k \), \( \{N_k; k = 0, 1, 2, \ldots\} \) is a Markov chain. The queueing model formulated here has both deterministic interarrival time and service time. The batch size \( X \) is determined by using the slotted ALOHA assumption for the up-links. Specifically, if \( n \) is the number of up-links, then the probability of \( j \) non-collided packet arrivals in a time slot is

\[
P\{X_k = j\} = \binom{n}{j} p^j (1 - p)^{n-j} = a_j
\]

with

\[
p = \frac{G}{n} e^{-G/n}.
\]
Here $G$ is the total traffic consisting of new packets, the traffic generated due to up-link collisions, and the traffic generated due to buffer overflow, and $p$ is the average number of packets arriving at the satellite for each up-link.

Since the expressions of the interesting measures studied in this paper will finally depend on the steady-state probabilities $\pi_i = \lim_{k \to \infty} P\{N_k = i\}$ of the number of packets at the satellite, finding $\pi_i$ is the key point to the performance analysis.

We start with some mathematically trivial or practically non-interesting cases and put them in the following remarks.

Remarks:

1. If there are more down-link transponders in the system than up-links, then the non-collided packets arriving at the satellite will be sent to their destinations in the same time slot without any extra delay. Thus, the steady-state probabilities of having $i$ packets at the satellite are the same as that of having $i$ non-collided packet arrivals in a time slot as long as the buffer size $B \geq n$; that is, $\pi_i = a_i$ for $i = 0, 1, \ldots, n$. For the case of $B < n$, up to $B$ non-collided packets will be accepted by the satellite in a time slot; the rest of the non-collided packets (if any) will be rejected due to a full buffer. Therefore, the steady-state probabilities are still the same as $a_i$ for $i < B$ and $\pi_B$ is the sum of the probabilities having at least $B$ non-collided arrivals; that is,

\[
\pi_i = a_i, \text{ for } i < B \quad \text{and} \quad \pi_B = \sum_{j=B}^{n} a_j.
\]  

2. A system which has at least as many up-links as transponders ($n \geq m$) is practically and mathematically interesting. For such a model, if the buffer size $B < m$, similar to the situation discussed in Remark 1, up to $B$ non-collided packets will be accepted at the satellite and sent to their destinations in the same time slot. Therefore, the steady-state probabilities are also given by (3).

A case of considerable practical importance which is studied in the remainder of this paper is the system with a smaller number of transponders than both up-links and buffer size; or $m < n$ and $m < B$. For the finite buffer case, according to whether or
not $B > n$, the steady-state equations for finding the steady-state probabilities $\pi_i$ are written, respectively, in the following.

For $m \leq B \leq n$:

\[
\pi_0 = a_0 \sum_{j=0}^{m} \pi_j,
\]

\[
\pi_i = a_i \sum_{j=0}^{m} \pi_j + \sum_{j=1}^{\min(i,B-m)} a_{i-j} \pi_{m+j}, \quad i = 1, 2, \ldots, B - 1,
\]

\[
\pi_B = \left( \sum_{i=B}^{n} a_i \right) \left( \sum_{j=0}^{m} \pi_j \right) + \sum_{j=1}^{B-m} \left( a_{B-j} + a_{B-j+1} + \cdots + a_n \right) \pi_{m+j}.
\]

For $m \leq n < B$:

\[
\pi_0 = a_0 \sum_{j=0}^{m} \pi_j, \quad \text{(4)}
\]

\[
\pi_i = a_i \sum_{j=0}^{m} \pi_j + \sum_{j=1}^{\min(n,B-m,i)} a_{i-j} \pi_{m+j}, \quad i = 1, 2, \ldots, n, \quad \text{(5)}
\]

\[
\pi_{i+n} = \sum_{j=0}^{\min(n,B-m-i)} a_{n-j} \pi_{m+i+j}, \quad i = 1, 2, \ldots, (B - n - 1), \quad \text{(6)}
\]

\[
\pi_B = \sum_{j=0}^{n-m} (a_{n-j} + a_{n-j+1} + \cdots + a_n) \pi_{m+B-n+j}. \quad \text{(7)}
\]

For the infinite buffer case, the steady-state equations will be given by (4), (5) and (6), in which (6) is valid for all $i \geq 1$.

### III The Steady-state Probabilities $\pi_i$

For the finite buffer case, a numerically stable algorithm derived based on the state-reduction method will be used for computing $\pi_i$. For details of the state-reduction method, one may refer to Grassmann, Taksar and Heyman [4] and Grassmann and
Heyman [5]. The rest of this section will be concerned with the infinite buffer model.

We define the generating function of the steady-state probabilities as

\[ P(z) = \sum_{i=0}^{\infty} \pi_i z^i. \]

The use of the generating function technique leads to the following expression:

\[ P(z) = \frac{A(z) \sum_{i=0}^{m-1} (z^m - z^i) \pi_i}{z^m - A(z)}, \]  \( \text{(8)} \)

where

\[ A(z) = \sum_{k=0}^{n} a_k z^k = [pz + (1 - p)]^n = E(z^X). \]

One may derive (8) using well known methods (for example, see Feller [3] or Boudreau et al. [1]). An equivalent form of (8) is

\[ P(z) = z^m \sum_{i=0}^{m-1} \left( z - z^m \right) \pi_i + z^m \frac{\sum_{i=0}^{m-1} (z^m - z^i) \pi_i}{z^m - A(z)}. \]

It should be noticed that the expression in (8) will not trivially or routinely lead to the determination of the equilibrium probabilities since the degree of the polynomial in the numerator is higher than the number of the zeros, inside or on the unit circle, of \( z^m - A(z) \). In order to obtain such an explicit formula, we separate the boundary probabilities \( \pi_i, 0 \leq i \leq m - 1 \), from the non-boundary probabilities \( \pi_i, i \geq m \), in the expression of the generating function. This will lead to explicit formulas for the non-boundary equilibrium probabilities, while the boundary equilibrium probabilities will be determined using another method. Specifically, it can be shown (for example, in [1]) that the generating function \( P(z) \) can be expressed in terms of the \( n - m \) zeros \( \theta_k, k = 1, 2, \ldots, n - m \), inside the unit circle, of the polynomial \( z^{n-m} - [p + (1 - p)z]^n \) in the form

\[ P(z) = z^m \sum_{i=0}^{m-1} \left( z - z^m \right) \pi_i + \frac{\pi_0 (1 - p)^n (1 - \theta_1 z)(1 - \theta_2 z) \cdots (1 - \theta_{n-m} z)}{(1 - p)^n (1 - \theta_1 z)(1 - \theta_2 z) \cdots (1 - \theta_{n-m} z)}. \]  \( \text{(9)} \)

This separation enables us to obtain explicit formulas for the non-boundary equilibrium probabilities and the probability of the system being empty. The condition \( P(1) = 1 \)
leads to the determination of $\pi_0$:

$$\pi_0 = (1 - p)^n(1 - \theta_1)(1 - \theta_2) \cdots (1 - \theta_{n-m}).$$

(10)

The expansion of (9) leads to

$$\pi_m = \frac{\pi_0}{(1 - p)^n} - \sum_{i=0}^{m-1} \pi_i,$$

(11)

and to the determination of $\pi_{m+i}$ for $i = 1, 2, \ldots$ by

$$\pi_{i+m} = \frac{\pi_0}{(1 - p)^n} \left[ C_1 \theta_1^i + C_2 \theta_2^i + \cdots + C_{n-m} \theta_{n-m}^i \right],$$

(12)

where

$$C_l = \frac{\theta_l^{n-m-1}}{\prod_{i \neq l}(\theta_l - \theta_i)}, \quad l = 1, 2, \ldots, n - m.$$  

(13)

Finally, $\pi_i$ for $i = 1, 2, \ldots, m - 1$ is determined by the corresponding steady-state equations and equations (11) and (12), with the result

$$\pi_i = a_i \prod_{k=1}^{n-m} (1 - \theta_k) + \frac{\pi_0}{(1 - p)^n} \sum_{k=0}^{i-1} a_k \left( \sum_{j=1}^{n-m} C_j \theta_j^{i-k} \right),$$

(14)

Equation (14) is also valid for $i = m$.

Now, we can see that for the infinite buffer model, the computation of the steady-state probabilities $\pi_i$ is trivial after the $n - m$ zeros $\theta_k$, inside the unit circle, of $h(z) = z^{n-m} - [p + (1 - p)z]^n$ are found. Since $h(z)$ is a polynomial, one may use any standard algorithm to find the required zeros. An alternative way is to use the following algorithm written specifically for this problem. The required number of iterations for finding a zero of $h(z)$ such that the difference between the computed zero and the precise one is smaller than $10^{-12}$ is less than 100 for a variety of cases chosen in our experiments. If $n - m = 1$, the root is positive; if $n - m = 2$, one of the roots is positive and the other is negative; and if $n - m \geq 3$, there are one or two real roots (according as $n - m$ is odd or even) and complex roots occurring in conjugate pairs. It can be shown using Rouché’s theorem that all the roots lie inside or on the circle going through the real
positive root. The following simple algorithm is recommended for computing the zeros:

For \( k = 1 \) to \( \left\lfloor \frac{n - m}{2} \right\rfloor + 1 \)

For \( 0 < r_0 < 1 \), let \( \alpha_k = e^{\frac{2k\pi}{n}} \) with \( j = \sqrt{-1} \) and \( z_0 = r_0 \alpha_k \)

\[
z_{i+1} = \frac{(1 - p)z_i^{m/n} + p + (1 - p)z_i}{\alpha_k^2} z_i^{m/n}
\]

Stop if \( |z_{i+1} - z_i| < \delta \)

IV The Validity of Using the Infinite Model

Let \( L_B \) and \( L_\infty \) be the average number of packets at the satellite for the finite buffer model and the infinite buffer model, respectively. The relative error of the average packets at the satellite between these two models is defined by

\[
e_B = \frac{|L_\infty - L_B|}{L_B}.
\]

The required buffer size for different values of parameters \( n, m \), and the offered traffic \( np \) has been computed by us such that the relative error \( e_B \) of the average packets in the system between these two models is less than \( 10^{-5} \). As we can see in Table 1 and Table 2, the required buffer size mainly depends on the traffic intensity \( \rho = np/m \), slightly depends on the ratio \( n/m \) of the up-links to the transponders, and is almost independent of the number of the transponders or up-links.

In the tables, an asterisk (*) means that the value of the traffic intensity \( \rho \) is not applicable. This is because the slotted ALOHA is used in this paper. The maximal \( p \) is reached at the total traffic equal to up-links: \( G = n \). In this case, \( p_{\text{max}} = e^{-1} \). Therefore, the maximum possible traffic intensity to the queueing system defined on the satellite is \( n/(me) \) and the parameters are called non-applicable if \( \rho > n/(me) \).
### Table 1. Required Buffer Sizes for Use of the Infinite Buffer Model ($m = 1$).

<table>
<thead>
<tr>
<th>$n/m$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
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<td>8</td>
<td>9</td>
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<td>10</td>
<td>9</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

### Table 2. Required Buffer Sizes for Use of the Infinite Buffer Model ($m = 4$).

<table>
<thead>
<tr>
<th>$n/m$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
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<tr>
<td>4</td>
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<td>6</td>
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<td>10</td>
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<td>10</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>

One might notice that the infinite buffer approximation is valid only for $\rho < 1$. This is because for an infinite model, the stability condition $\rho = np/m < 1$ is needed. Otherwise the average number of the non-collided packet arrivals in a time slot will
be larger than the number of the transponders and the system becomes unstable. We now show that, for $\rho \geq 1$, the average number of packets rejected per time slot due to a full buffer is the difference, $np - m$, between the offered traffic and the number of transponders.

Let $M_k$ be the number of buffer spaces available at the satellite at time $t_k$ for $k = 1, 2, \ldots$. Then $M_k = B - N_k$. Let $p_j$ be the limiting probability of the number of buffer spaces available at the satellite:

$$p_j = \lim_{k \to \infty} P\{M_k = j\} = \pi_{B-j}$$

and let $G(z)$ be the generating function of $p_j$:

$$G(z) = \sum_i p_i z^i.$$  

For the infinite buffer case, this expression as the limit of the finite case can be written as

$$G(z) = \frac{H(z)}{z^{n-m} - [p + (1-p)z]^n},$$

where

$$H(z) = z^{n-m} \left( \sum_{j=0}^{n-m-1} a_{j+m+1} \sum_{l=0}^j p_l \right) - \sum_{i=0}^{n-m-1} \left( \sum_{l=0}^i p_l a_{l+n-i} \right) z^i,$$  \quad (15)

which is a polynomial of degree $n - m$.

It follows from using $G(1) = 1$ and L'Hôpital's Rule that $H'(1) = np - m$. On the other hand, by using (15),

$$H'(1) = \sum_{l=0}^{n-m-1} p_l \sum_{j=l}^{n-m-1} [(n-m)a_{j+m+1} - ja_{l+n-j}].$$

Notice that we can write

$$\sum_{j=l}^{n-m-1} ja_{l+n-j} = \sum_{i=1}^{n-m-l} (n-m-i)a_{m+l+i}$$

and

$$\sum_{j=l}^{n-m-1} (n-m)a_{j+m+1} = \sum_{i=1}^{n-m-l} (n-m)a_{m+l+i}.$$
So

\[ H'(1) = \sum_{l=0}^{n-m-1} p_l \sum_{i=1}^{n-m-l} i a_{m+l+i} . \]

Let \( S_r \) be the average number of packets rejected per time slot due to a full buffer. Then it is easily shown that

\[ S_r = \sum_{j=B-(n-m-1)}^{B} \pi_j \left[ \sum_{i=B-j+m+1}^{n} (i - B - m + j) a_i \right]. \]  \hspace{1cm} (16)

For \( \rho < 1 \), \( S_r \to 0 \) as \( B \to \infty \). For \( \rho \geq 1 \), use \( p_j = \pi_{B-j} \) and take the limit as \( B \) goes to infinity. A comparison with (16) shows that \( S_r = H'(1) \). Thus \( S_r = np - m \) for \( \rho \geq 1 \).

V Performance Measures

The system throughput \( S \) is defined as the difference between the average number \( np \) of packets arriving without collision on the up-links and the average number \( S_r \) rejected due to a full buffer, so that \( S = np - S_r \).

For a fixed offered traffic load \( np \), the system throughput \( S \) increases as the buffer size increases. If the traffic intensity \( \rho = np/m < 1 \), the system throughput increases to \( np \), which is the number of non-collided packets arriving at the satellite in a time slot. If \( \rho > 1 \), the system throughput approaches \( m \), which is the number of transponders, as the buffer size increases to infinity. The dependence of throughput on buffer size \( B \) and offered traffic \( np \) is shown in Figure 1 and Figure 2.

The buffer overflow probability \( P_{of} \) is evaluated by the ratio of the rejected to the offered traffic:

\[ P_{of} = \frac{S_r}{np} . \]

How the buffer overflow probability changes with the buffer size and the offered traffic is shown in Figure 3. As we expect, the buffer overflow probability decreases as the buffer size increases.

The average number \( L \) of packets at the satellite is given in terms of \( \pi_i \):

\[ L = \sum_{i} i \pi_i . \]
For an infinite buffer model, the expression of this number only depends on the zeros \( \theta_1, \theta_2, \ldots, \theta_{n-m} \). In fact,

\[
L = P'(1) = \frac{2mnp - n(n + 1)p^2 - m(m - 1) + \sum_{i=0}^{m-1}\pi_i[m(m-1) - i(i-1)]}{2(m-np)},
\]

where \( \pi_0, \pi_1, \ldots, \pi_{m-1} \) are determined according to (10) and (14).

When the traffic intensity \( \rho = np/m < 1 \), the average number of packets at the satellite increases to a finite number as the buffer size increases to infinity; but when \( \rho \geq 1 \), the average number of packets at the satellite increases to infinity as the buffer size goes to infinity. Figure 4 shows the relationship.

A non-collided packet may not be accepted by the satellite due to a full buffer. The average delay \( W_l \) of a packet at the satellite is defined as the time delay on the satellite of the last packet in a successful arrival batch of non-collided packets. The assumption implied by this definition is that there is at least one packet in the arrival batch. Mathematically, if \( W \) is the average possible waiting time of the last packet in a arrival batch at the satellite including the situation there is no packet in the batch, then the average delay \( W_l \) of the last packet in a successful arrival batch of the non-collided packets is the conditional waiting time conditioned on there being at least one packet in the arrival batch:

\[
W_l = E[W \mid \text{at least one packet in the arrival batch}] = \frac{E[W]}{1 - \pi_0}

= \frac{1}{1 - \pi_0} \left[ \sum_{k=1}^{m} \pi_k + 2 \sum_{k=m+1}^{2m} \pi_k + \cdots + b_0 \sum_{k=(b_0-1)m+1}^{b_0m} \pi_k + (b_0 + 1) \sum_{k=b_0m+1}^{b_0m+r_0} \pi_k \right],
\]

where \( b_0 \) and \( r_0 \) are non-negative integers such that \( B = b_0m + r_0 \) with \( 0 \leq r_0 < m \).

For an infinite buffer model with \( \rho < 1 \), it can be shown by using (11) and (12) that

\[
W_l = \frac{1}{1 - \pi_0} \left[ \prod_{k=1}^{n-m} (1 - \theta_k) + \sum_{k=1}^{n-m} \frac{2C_k\theta_k}{(1 - \theta_k)(1 - \theta_k^m)^2} \right],
\]

where \( C_k, k = 1, 2, \ldots, n - m \), are defined by (13).
As shown in Figure 5, the average delay of a packet at the satellite increases when either the buffer size or the offered traffic increases.

The retransmission of a packet is required either by an up-link collision or by a full buffer. The probability of a successful transmission is given by

$$e^{-G/n}(1 - P_{of}).$$

Therefore the average number $N_r$ of retransmissions is computed according to

$$N_r = \frac{1 - e^{-G/n}(1 - P_{of})}{e^{-G/n}(1 - P_{of})}.$$ 

If $\rho < 1$, the limiting value of the number of retransmissions as $B$ goes to infinity is

$$\lim_{B \to \infty} N_r = \frac{1 - e^{-G/n}}{e^{-G/n}} = e^{G/n} - 1.$$ 

However, if $\rho > 1$, the limiting value of the number of retransmissions as $B$ goes to infinity is

$$\lim_{B \to \infty} N_r = \frac{1 - e^{-G/n} \left( \frac{m}{np} \right)}{e^{-G/n} \left( \frac{m}{np} \right)} = \rho e^{G/n} - 1.$$ 

It can be seen in Figure 6 that the system performance degenerates for a very small buffer size.

The total transmission delay $D$ consists of two parts, the transmission delay $D_t$ and the retransmission delay $D_r$. That is,

$$D = D_t + D_r,$$

where

$$D_t = 1 + R + W_t$$

includes the round trip propagation delay $R$ and the delay at satellite, and

$$D_r = N_r \left( 1 + D_a + \frac{K + 1}{2} \right)$$

is the multiple of the average retransmission number and the delay per one retransmission including the acknowledgement delay $D_a$ and the average delay for resending the
rejected packet. Except for $W_l$ and $N_r$, all parameters for computing the total delay can be treated as constants. The total delay described in Figure 7 was computed according to $D = W_l + 10N_r$, which consists of two parts, from the delay of the packet at the satellite and the number of retransmissions.

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**References**


Figure 1. System Throughput vs Buffer Size \((n = 9, m = 3)\)

\[ S \]

\[ np = n/e = 3.31 \]
\[ (\rho = 1.103) \]

\[ np = 2.75 \]
\[ (\rho = 0.92) \]

\[ np = 2.50 \]
\[ (\rho = 0.83) \]

\[ np = 2.25 \]
\[ (\rho = 0.75) \]

\[ np = 2.00 \]
\[ (\rho = 0.67) \]
Figure 2. System Throughput vs the Offered Traffic \((n = 9, m = 3)\)

From top to bottom:
- \(B = 50\)
- \(B = 7\)
- \(B = 5\)
- \(B = 3\)
- \(B = 1\)
Figure 3. Buffer Overflow Probability vs Buffer Size ($n = 9$, $m = 3$)

$P_{of}$

\[ np = 3.31 \quad (\rho = 1.103) \]

\[ np = 3.00 \quad (\rho = 1.00) \]

\[ np = 2.50 \quad (\rho = 0.83) \]

\[ np = 2.00 \quad (\rho = 0.67) \]
Figure 4. Average number of packets at the satellite vs buffer size ($n = 9$, $m = 3$)

$L$

\[ np = 2.75 \]  
\[ (\rho = 0.92) \]

\[ np = 2.5 \]  
\[ (\rho = 0.83) \]

\[ np = 2.25 \]  
\[ (\rho = 0.75) \]

\[ np = 2.0 \]  
\[ (\rho = 0.67) \]

\[ np = 1.0 \]  
\[ (\rho = 0.33) \]

$B$
Figure 5. Average delay at the satellite vs the buffer size $(n = 9, \ m = 3)$

From top to bottom:

- $np = 3.3 \ (\rho = 1.10)$
- $np = 3.0 \ (\rho = 1.00)$
- $np = 2.5 \ (\rho = 0.83)$
- $np = 2.0 \ (\rho = 0.67)$
- $np = 1.0 \ (\rho = 0.33)$
Figure 6. Average number of retransmissions vs the buffer size \((n = 9, m = 3)\)

\(N_r\)

\[ np = 3.31 \ (\rho = 1.103) \]

\[ np = 3.00 \ (\rho = 1.00) \]

\[ np = 2.75 \ (\rho = 0.92) \]

\[ np = 2.50 \ (\rho = 0.83) \]

\[ np = 2.00 \ (\rho = 0.67) \]
Figure 7. Total transmission delay vs the buffer size \( (n = 9, m = 3) \)

\[ D \]

\[ np = 3.31 \ (\rho = 1.103) \]

\[ np = 3.0 \ (\rho = 1.00) \]

\[ np = 2.75 \ (\rho = 0.92) \]

\[ np = 2.5 \ (\rho = 0.83) \]

\[ np = 2.0 \ (\rho = 0.67) \]

\[ np = 1.5 \ (\rho = 0.50) \]