

CHAPTER 1, QUESTION 7

7. Let D be an integral domain. Let $u \in U(D)$. Let I be an ideal of D that contains u . Prove that $I = D$.

Solution. As $u \in U(D)$ there exists $v \in D$ such that

$$uv = 1.$$

Further, as $u \in I$, $v \in D$, and I is an ideal, we have

$$uv \in I.$$

Thus

$$1 \in I.$$

Let $d \in D$. As I is an ideal $d \cdot 1 \in I$. Thus $d \in I$ so $D \subseteq I$. But I is an ideal of D so that $I \subseteq D$. Hence $I = D$. ■

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