

CHAPTER 1, QUESTION 11

11. Give an example to show that if A and B are ideals of an integral domain D then $A \cup B$ may not be an ideal of D .

Solution. Let $D = \mathbb{Z}$, $A = \langle 3 \rangle$, $B = \langle 5 \rangle$. Then

$$\begin{aligned} 3 &\in A \subset A \cup B, \\ 5 &\in B \subset A \cup B. \end{aligned}$$

However

$$5 - 3 = 2 \notin A \cup B,$$

so that $A \cup B$ is not an ideal of D . ■

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