Uplink Power Adjustment in Wireless Communication Systems: A Stochastic Control Analysis

Minyi Huang, *Member, IEEE*, Peter E. Caines, *Fellow, IEEE*, and Roland P. Malhamé, *Member, IEEE*

***Abstract*—This paper considers mobile to base station power control for lognormal fading channels in wireless communication systems within a centralized information stochastic optimal control framework. Under a bounded power rate of change constraint, the stochastic control problem and its associated Hamilton–Jacobi–Bellman (HJB) equation are analyzed by the viscosity solution method; then the degenerate HJB equation is perturbed to admit a classical solution and a suboptimal control law is designed based on the perturbed HJB equation. When a quadratic type cost is used without a bound constraint on the control, the value function is a classical solution to the degenerate HJB equation and the feedback control is affine in the system power. In addition, in this case we develop approximate, but highly scalable, solutions to the HJB equation in terms of a local polynomial expansion of the exact solution. When the channel parameters are not known *a priori*, one can obtain on-line estimates of the parameters and get adaptive versions of the control laws. In numerical experiments with both of the above cost functions, the following phenomenon is observed: whenever the users have different initial conditions, there is an initial convergence of the power levels to a common level and then subsequent approximately equal behavior which converges toward a stochastically varying optimum.**

***Index Terms*—Dynamic programming, Hamilton–Jacobi–Bellman (HJB) equations, lognormal fading channels, power control, quality of service.**

I. Introduction

POWER control in cellular telephone systems is important at the user level in order to both minimize energy requirements and to guarantee constant or adaptable quality of service (QoS) in the face of telephone mobility and fading channels. This is particularly crucial in code division multiple access (CDMA) systems where individual users are identified not by a particular frequency carrier and a particular frequency content, but by a wideband signal associated with a given pseudorandom number code. In such a context, the received signal of a given user at the base station views all other incell user signals, as well as other cell signals arriving at the base station, as interference or noise, because they both degrade the decoding process of identifying and extracting a given user's signal. Thus, it becomes crucial that individual mobiles emit power at a level which will insure adequate signal-to-interference ratio (SIR) at the base station. More specifically, excess levels of signalling from a given mobile will act as interference on other mobile signals and contribute to an accelerated depletion of cellular phone batteries. Conversely, low levels of signalling will result in inadequate QoS. In fact, tight power control is also indirectly related to the ability of the CDMA base station to accommodate as many users as possible while maintaining a required QoS [41].

There has been a rich literature on the topic of power control. Previous attempts at capacity determination in CDMA systems have been based on a "load balancing" view of the power control problem [41]. This reflects an essentially static or at best quasi-static view of the power control problem which largely ignores the dynamics of channel fading as well as user mobility. In essence, in this formulation power control at successive sampling time points is viewed as a pointwise optimization problem with total statistical independence assumed between the variables (control or signal) at distinct time points. For the computation of various static optimal power levels, distributed algorithms have been developed in [29], [42] with constant channel gain. In a deterministic framework, [36], [37] present an attempt at reintroducing dynamics into the analysis, at least insofar as convergence analysis to the static pointwise optimum is concerned. This is achieved by recognizing that power level set points dictated by the base station to the mobile can only increase or decrease by fixed amounts. In [1], power control is considered for a CDMA system in which an SIR based utility function is assigned to each user; this gives rise to a game theoretic formulation to power optimization. For spread spectrum wireless networks, Hanly and Tse studied power control and its relationship with system capacity [14]. In the stochastic framework, attempts at recognizing the time correlated nature of signals are made in [27], where blocking is defined, not as an instantaneous reaching of a global interference level but via the sojourn time of global interference above a given level which, if sufficiently long, induces blocking. The resulting analysis employs the theory of level crossings. In [24], the authors proposed power control methods for Rayleigh fading channels using outage probability specifications. Downlink power control for fading channels is studied in [3] by a heavy traffic limit where averaging methods are used. Recent work on dynamic power control with stochastic channel variation can be found in [5], [35], and [38], and power compensation for lognormal shadowing effects is considered in [35] and [38].

Manuscript received October 5, 2002; revised August 27, 2003 and April 31, 2004. Recommended by Associate Editor D. Li. This work was supported by the NCE-MITACS Program 1999–2002 and by NSERC Grants 1329-00 and 1361-00.

M. Huang was with the Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 2A7, Canada. He is now with the Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, 3010 VIC, Australia (e-mail: m.huang@ee.mu.oz.au).

P. E. Caines is with the Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 2A7, Canada, and is affiliated with GERAD, Montreal (e-mail: peterc@cim.mcgill.ca).

R. P. Malhamé is with the Department of Electrical Engineering, École Polytechnique de Montréal, Montreal, QC H3C 3A7, Canada, and is affiliated with GERAD, Montreal (e-mail: roland.malhame@polymtl.ca).

Digital Object Identifier 10.1109/TAC.2004.835388

In contrast to those papers, the modeling and analysis of power control strategies investigated here employ wireless models which are time-varying and subject to fading. In particular, the dynamic model for power loss expressed in decibels (dBs) is a linear stochastic differential equation whose properties model the long-term fading effects due to: 1) reflection power loss, and 2) power loss due to long distance transmission of electromagnetic waves over large areas [6], [8]. This gives rise to power loss trajectories which are log-normally distributed. Lognormal power loss models are justified by experimental data [30], [33].

Concerning wireless channel modeling, we note that radio channels experience both large-scale fading (long-term effects) and small-scale fading (short-term effects). Large-scale fading is modeled by lognormal distributions and small-scale fading can be modeled by Rayleigh or Rician distributions [33]. In general, large-scale fading and small-scale fading are considered as superimposed and may be treated separately [25], [33], [44]. In this paper, we only consider dynamic modeling of the large-scale fading and its transmission power compensation.

Motivated by current technology, we propose a (bounded) rate based power control model for the power adjustment of lognormal fading channels and then different performance functions are introduced. The structure of each of the performance functions is related to the system SIR requirements. We do not make direct use of the SIR or other related quantities such as the bit error rates (BERs) [9] or outage probabilities in the definition of the performance function; instead, we use a loss function integrated over time which depends upon the factors determining the SIRs and the power levels. By this means, we will be able to avoid certain technical difficulties in the analysis and computation of the control laws. Our current analysis of the optimal control law of each individual user involves centralized information, i.e., the control input of each user depends on the state variable of all the users. It is of significant interest to investigate the feasibility of decentralized control under fading channels since this may substantially reduce the system complexity for practical implementation of the control laws. Indeed, our stochastic control framework can be combined with certain approximation techniques to give relatively simple (partially decentralized) control laws for practical systems with many users; see [22].

The paper is organized as follows. In Section II, we propose an optimal control formulation for CDMA power adjustment which includes a fading channel model, a power control model and a performance function which is intended to reflect power minimization objectives under SIR constraints. In this section, following [36], [37], and taking into account existing wireless technology [32], we introduce a "rate based" power set point bounded control input model. An important consequence of the existence of a bound on the rate of change of mobile power, is that successive uplink power adjustments can no longer be considered as a sequence of independent pointwise optimization problems (the currently prevailing telecommunications view). In Section III, for an isolated cell we analyze the optimal stochastic control and introduce the associated degenerate HJB equation. The solution of the HJB equation is sought in a viscosity solution framework. Section IV is devoted to the suboptimal approximation of the value function by suboptimal classical solutions, and Section V contains numerical solutions to the approximating HJB equation and simulations of the suboptimal control laws. In Section VI, we remove the bound constraint on the control input by introducing a quadratic type cost function which leads to an analytic solution for the feedback control law. Based on this analytic solution, further approximations to the control law are considered. These approximations are of particular interest from an engineering point of view because they make computations of control laws feasible for large systems. Moreover, for systems with a large number of users the linear quadratic approach in Section VI is also useful in developing decentralized or partially decentralized control laws by appropriately approximating the total interference one user receives from all other users; this may be addressed in the so-called individual versus mass framework [21]. In the main part of the analysis in this paper, we have assumed known channel dynamics. When the channel parameters are unknown, one can employ the parameter estimation scheme in Appendix C to get online estimates of the channel parameters and construct adaptive versions of the control laws.

In Sections V and VI, in the numerical experiments in both the bounded control and quadratic type cost function cases, the following phenomenon is observed: whenever the users have different initial conditions there is an initial convergence of the power levels to a common level and then subsequent approximately equal behavior which converges toward a pointwise (stochastically varying) optimum. This phenomenon constitutes a notable cooperative behavior of users when cooperation is induced by the centralized optimal control formulation adopted in this paper. Finally, the conclusion outlines future work.

II. Optimal Control Formulation

A. Channel Model

There has been an extensive literature on modeling of radio propagation. Generally, radio channels experience both small-scale fading (short-term effects, modeled by Rayleigh or Rician distributions) and large-scale fading (long-term effects, modeled by lognormal distributions), which result in random fluctuations of received power for mobile users. In general, the two different fading effects are understood as superimposed and can be treated separately due to the different mechanisms from which they are generated [25], [33]. Methods to mitigate the impairments of large-scale fading and small-scale fading are quite different. Indeed, small-scale (with a time scale of milliseconds) fading is caused by multipath replicas of the same signal which, in view of their respective phase shifts, can interact either constructively, or destructively. It is a problem which can be addressed via the so-called diversity techniques (see [23] and [33]). Large-scale (with a time scale of hundreds of milliseconds) fading is caused by shadowing effects due to buildings and moving obstacles, such as trucks, partially blocking or deflecting mobile or base station signals.

Practical power control algorithms can efficiently compensate for large-scale fading but cannot effectively cope with small-scale fading [15]; the more effective techniques to combat small-scale fading include antenna arrays and coding, etc. [44]. For these reasons, in the subsequent analysis we only deal with large-scale (lognormal) fading, and small-scale fading will not be in the scope of our work. We note that in certain environments the small-scale component may play an increasingly important role for channel modeling.

For the discrete time scenario, based on Gudmundson's experimental measurements, first order autoregressive (AR) innovation models have been widely used for dynamic modeling of large-scale fading (see, e.g., [13], [38], and [40]); in these AR innovation models, large-scale fading is described in dB as Gaussian Markov processes. In this paper we follow the stochastic differential equation approach proposed by Charalambous and Menemenlis [6] for the dynamic modeling of large-scale fading. This is intended as a general setup for continuous time modeling of the spatio-temporal correlation of large-scale fading taking into account user mobility.

Let $x_i(t)$, $1 \le i \le n$, denote the attenuation (expressed in dBs and scaled to the natural logarithm basis) at the instant t of the power of the ith mobile user of a network and let $\alpha_i(t) = e^{x_i(t)}$ denote the actual attenuation. Based on the work in [6], we model the power attenuation dynamics by the so-called mean reverting Ornstein–Ulenbeck process

$$dx_i = -a_i(x_i + b_i)dt + \sigma_i dw_i, \qquad t \ge 0, \quad 1 \le i \le n \tag{1}$$

where n denotes the number of mobile users, $\{w_i, 1 \le i \le n\}$ are n independent standard Wiener processes, and $x_i(0)$, $1 \le i \le n$, are mutually independent Gaussian random variables which are also independent of the Wiener processes. In (1), $a_i > 0$, $b_i > 0$, and $\sigma_i > 0$, $1 \le i \le n$. The first term in (1) implies a long-term adjustment of x_i toward the long-term mean $-b_i$, and a_i is the speed of the adjustment. The constant b_i is interpreted as the average large-scale path loss [6]. In typical mobile communication scenarios, the change of the channel attenuation is primarily due to the spatial variation of the lognormal shadow fading component, and the effect of user-base distance change is usually far less significant; see [38], [40], and a simple numerical example for a macrocell in [16]. As a result, the attenuation manifests itself as oscillations around a constant level during the service session. For this reason, in the channel modeling it is plausible to set b_i as a fixed constant when user mobility is taken into account.

In (1), the parameters a_i, σ_i indicate the variation rate of the channel gain, and are related to the user mobility level and the volatility of the underlying lognormal shadowing effects. For statistical characterization of shadowing effects, the interested reader is referred to [33].

B. Rate-Based Power Control

Currently, the power control algorithms employed in the mobile telephone domain use gradient type algorithms with bounded step size [14], [32], [39]. This is motivated by the fact that cautious algorithms are sought which behave adaptively in a communications environment in which the actual position of the mobile and its corresponding channel properties are unknown and varying. A limited step size is also desirable when mobile power levels are close to optimum.

We model the adaptive stepwise adjustments of the (sent) power p_i (i.e., that sent in practice by the ith mobile) by the so-called rate adjustment model [17], [18]

$$dp_i = u_i dt, \qquad t \ge 0, \quad |u_i| \le u_{i_{\max}}, \qquad 1 \le i \le n \tag{2}$$

where the bounded input u_i controls the size of increment dp_i at the instant t. Without loss of generality we set $u_{i_{\max}} = 1$. The adaptive nature of practical rate adjustment control laws is replaced here by an optimal control calculation based on full knowledge of channel parameters a_i, b_i, and σ_i, $1 \le i \le n$. In the intended practical implementation of our solution these parameters would be replaced by online estimates; see Appendix C for the parameter estimation algorithm. Write

$$x = [x_1, \ldots, x_n]^\tau \quad \alpha = [e^{x_1}, \ldots, e^{x_n}]^\tau$$
$$p = [p_1, \ldots, p_n]^\tau \quad u = [u_1, \ldots, u_n]^\tau.$$

We note that the rate adjustment model (2) is similar to the discrete-time up/down power control scheme proposed in [35] where the power at the next time instant is calculated from the current power level and a bounded additive tuning term which is optimized by a statistical linearization technique employing the current power, the channel state and a target SIR.

C. Performance Function

Let $\eta > 0$ be the constant system background noise intensity which is assumed to be the same for all n mobile users in a network. Then, in terms of the power levels $p_i \ge 0$, $1 \le i \le n$, and the channel power attenuations α_i, $1 \le i \le n$, the so-called SIR for the ith mobile is given by $\Gamma_i = \alpha_i p_i / (\sum_{j \ne i}^n \alpha_j p_j + \eta)$, $1 \le i \le n$. A standard communications QoS constraint is to require that

$$\Gamma_i \ge \gamma_i > 0, \qquad 1 \le i \le n \tag{3}$$

where γ_i, $1 \le i \le n$, is a prescribed set of individual SIR's. The constraints (3) are equivalent to the linear constraints $\alpha_i p_i \ge \gamma_i (\sum_{j \ne i}^n \alpha_j p_j + \eta)$, which, in turn, are equivalent to $(1 + \gamma_i)\alpha_i p_i \ge \gamma_i (\sum_{j=1}^n \alpha_j p_j + \eta)$ and, hence, to

$$\Gamma_i' = \frac{\alpha_i p_i}{\sum_{j=1}^n \alpha_j p_j + \eta} \ge \mu_i, \qquad 1 \le i \le n \tag{4}$$

where $\mu_i \triangleq (\gamma_i / (1 + \gamma_i)) > 0$, $1 \le i \le n$. It is easily verified that there exists at least one positive power vector p satisfying (4) if and only if

$$0 < \sum_{i=1}^n \mu_i < 1. \tag{5}$$

A straightforward way to formulate the optimization problem would be to seek control functions which yield the minimization of the integrated power $\int_0^T \sum_{i=1}^n p_i(t) dt$, subject to the constraints (4), (5) at each instant t, $0 \le t \le T$. First, however, consider the pointwise global minimization of the summed power $\sum_{i=1}^n p_i$ under the inequality constraints (4), (5) and the constraints $p_i \ge 0$, $1 \le i \le n$. Taking the n inequalities in (4) as equalities and taking into account the constraint (5), we get a positive power vector $p^0 = (p_1^0, \ldots, p_n^0)^\tau$ given by $p_i^0 = \mu_i \eta / (\alpha_i(1 - \sum_{i=1}^n \mu_i))$, $1 \le i \le n$. It turns out that p^0 is the unique positive vector minimizing $\sum_{i=1}^n p_i$ under constraints (4), (5). Furthermore, it can be shown [37] that any nontrivial local perturbation of p^0 to a vector p which also satisfies the constraints results in a strict increase of each component p_i^0. Hence, such a p^0 is a local (linear inequality constrained) minimum which is also a global (linear inequality constrained) minimum. In other words, provided (5) holds, the solution to

$$\text{minimize} \quad \sum_{i=1}^n p_i, \qquad p_i \ge 0 \tag{6}$$

subject to (4) is the unique solution to

$$\frac{\alpha_i p_i}{\sum_{j=1}^n \alpha_j p_j + \eta} = \mu_i, \qquad 1 \le i \le n. \tag{7}$$

Hence, it is well motivated to replace such a pointwise constrained deterministic optimization problem with the corresponding unconstrained deterministic penalty function optimization problem

$$\text{minimize} \quad \sum_{i=1}^n \left[\alpha_i p_i - \mu_i \left(\sum_{j=1}^n \alpha_j p_j + \eta \right) \right]^2 + \lambda \sum_{i=1}^n p_i \tag{8}$$

over $p_i \ge 0$, $1 \le i \le n$, where $\lambda \ge 0$. However, because the power vector is a part of the stochastic channel-power system state with dynamics (1), (2) and full state (x, p), it is impossible to instantaneously minimize (8) via $u(t)$ at all times t. Hence, over the interval $[0, T]$, we employ the following averaged integrated loss function:

$$E \int_0^T \left\{ \sum_{i=1}^n \left[\alpha_i p_i - \mu_i \left(\sum_{j=1}^n \alpha_j p_j + \eta \right) \right]^2 + \lambda \sum_{i=1}^n p_i \right\} dt \tag{9}$$

subject to (1) and (2), where $\lambda \ge 0$. It is an extra property of the loss function (9) corresponding to (6) and (7) that overshoots near the optimum are penalized.

Clearly, in the cost function (9), the first term of the integrand is related to the instantaneous SIR in an indirect way. We note that if the SIR term Γ_i in (3) were to be employed directly in the cost function it would cause a potential zero division problem and present more analytic difficulties, since in our current formulation we do not add hard constraints to ensure positivity of the powers.

In a practical implementation, the power of each user should remain positive. To meet such a requirement, we can choose appropriate control models. For instance, one might choose the control model $dp_i = u_i p_i dt$, $1 \le i \le n$, with a positive initial power vector. However, this and related setups may deviate significantly from the technology actually in use. Instead, we use the rate based control model and the loss function introduced previously. By choosing a small weighting coefficient λ and increasing the upper bound $u_{i_{\max}}$ for the control input, we can guarantee that the optimally controlled power process $\widehat{p}$ obtained in the stochastic optimal control framework takes a negative value with only a small probability. For a better understanding of this point, we consider the ideal powers for minimizing the integrand of (9). For a fixed time period, we assume the attenuations to be constants and write

$$\begin{pmatrix} q_1 \\ q_2 \\ \cdots \\ q_n \end{pmatrix} = \begin{pmatrix} 1-\mu_1 & -\mu_1 & \cdots & -\mu_1 \\ -\mu_2 & 1-\mu_2 & -\mu_2 & \cdots \\ \cdots & & & \cdots \\ -\mu_n & -\mu_n & \cdots & 1-\mu_n \end{pmatrix} \begin{pmatrix} \alpha_1 p_1 \\ \alpha_2 p_2 \\ \cdots \\ \alpha_n p_n \end{pmatrix}. \tag{10}$$

Setting $q = (q_1, \ldots, q_n)^\tau$, we write the integrand in (9) as $F(q) = \sum_{i=1}^n (q_i - \mu_i \eta)^2 + \lambda \sum_{i=1}^n \beta_i q_i$, where the coefficients β_i are determined from (10). The minimum of $F(q)$ is attained at $\overline{q} = (\overline{q}_1, \ldots, \overline{q}_n)^\tau$, where

$$\overline{q}_i = \frac{2\mu_i \eta - \lambda \beta_i}{2} \qquad 1 \le i \le n. \tag{11}$$

Thus, when the attenuations are fixed and $0 \le \lambda \ll 1$, (11) gives a positive vector $\overline{q}$. By straightforward calculation it can be further shown that under the condition (5), the coefficient matrix in (10) has an inverse with all positive entries and, therefore, we can obtain a positive power vector p^0 from $\overline{q}$ using (10). Although p^0 cannot be realized instantaneously by a control input, the optimal control will try to track p^0. Whenever the power of the system deviates from p^0, a greater penalty results. In such a manner the optimal control will try to steer the optimally controlled power to be positive with a large probability.

Throughout this paper, we assume the following assumption holds

H1) The positive constants μ_i, $1 \le i \le n$, in the cost function (9) satisfy the inequality (5), i.e., $0 < \sum_{i=1}^n \mu_i < 1$. $\square$

III. Analysis of the Optimal Control

In the following, we analyze the optimal control problem in terms of the state vector (x, p); this facilitates the definition of the value function v since x_i is defined on $\mathbb{R}$, while α_i is only defined on $\mathbb{R}_+$, $1 \le i \le n$. Further define

$$f(x) = \begin{pmatrix} -a_1(x_1 + b_1) \\ \vdots \\ -a_n(x_n + b_n) \end{pmatrix} \quad H = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n \end{pmatrix}$$
$$G = \begin{pmatrix} H \\ 0_{n \times 0} \end{pmatrix} \tag{12}$$

and set $z = (x^\tau, p^\tau)^\tau$, $\psi = (f^\tau, u^\tau)^\tau$. We write (1) and (2) in the vector form

$$dz = \psi dt + G dw \qquad t \ge 0 \tag{13}$$

where w is an $n \times 1$ standard Wiener process determined by (1). In the analysis, we will denote the state variable either by (x, p) or by z, or in a mixing form, when it is convenient. We also rewrite the integrand in (9) in terms of (x, p) as

$$L(z) = L(x,p) = \sum_{i=1}^n \left[e^{x_i} p_i - \mu_i \left(\sum_{j=1}^n e^{x_j} p_j + \eta \right) \right]^2 + \lambda \sum_{i=1}^n p_i.$$

As is stated in Section II-A, the initial value of x at $t=0$ is independent of the $n\times 1$ Wiener process; we make the additional assumption that $p|_{t=0}$ is deterministic.

The admissible control set is specified as $\mathcal{U} = \{u(\cdot)|u$ adapted to $\sigma(x_s, s \le t)$, and$u(t) \in U \triangleq [-1,1]^n$, $\forall 0 \le t \le T\}$. Define $\mathcal{L} = \{u(\cdot)|u$ adapted to $\sigma(x_s, s \le t)$, and $E\int_0^T |u_t|^2 dt < \infty,\}$. If we endow $\mathcal{L}$ with an inner product $\langle u, v\rangle = E\int_0^T u^\tau v ds$, for $u,\ v \in \mathcal{L}$, then $\mathcal{L}$ constitutes a Hilbert space. By the previous inner product we have on $\mathcal{L}$ an induced norm $\|\cdot\|$, under which $\mathcal{U}$ is a bounded, closed and convex subset of $\mathcal{L}$. Finally, the cost associated with the system (13) and a control u is specified to be $J(s,x,p,u) = E[\int_s^T L(x_t,p_t)dt|x_s = x, p_s = p]$, where $s \in [0,T]$ is taken as the initial time of the system; further we set the value function

$$v(s,x,p) = \inf_{u\in\mathcal{U}} J(s,x,p,u)$$

and simply write $J(0,x,p,u)$ as $J(x,p,u)$. Throughout this paper, we use x_i (or p_i) with an integer subscript $1 \le i \le n$ to denote the ith entry in the vector x (or p) respectively, and x or p associated with a real valued subscript $t \ge 0$ or $s \ge 0$ (e.g., x_s) to denote the value of the vector process at time t or s.

Theorem 3.1: There exists a unique $\hat{u} \in \mathcal{U}$ such that $J(x_0,p_0,\hat{u}) = \inf_{u\in\mathcal{U}} J(x_0,p_0,u)$, where (x_0,p_0) is the initial state at time $s=0$, and uniqueness holds in the following sense: if $\tilde{u} \in \mathcal{U}$ is another control such that $J(x_0,p_0,\tilde{u}) = J(x_0,p_0,\hat{u})$, then $P_\Omega(\tilde{u}_s \ne \hat{u}_s) > 0$ only on a set of times $s \in [0,T]$ of Lebesgue measure zero, where Ω is the underlying probability sample space.

Proof: See Appendix A. □

Proposition 3.2: The value function v is continuous on $[0,T]\times\mathbb{R}^{2n}$, and furthermore

$$v(t,x,p) \le C\left(1 + \sum_{i=1}^n p_i^4 + \sum_{i=1}^n e^{4x_i}\right) \tag{14}$$

where $C > 0$ is a constant independent of (t,x,p).

Proof: The continuity of v can be established by continuous dependence of the cost on the initial condition of (13). Inequality (14) is obtained by a direct estimate of the cost function. □

We see that in (13) the covariance matrix GG^τ is not of full rank. In general, under such a condition the corresponding stochastic optimal control problem does not admit classical solutions due to the degenerate nature of the arising HJB equations. Thus we adopt viscosity solutions.

Definition 3.3: [43] $\overline{v}(t,z) \in C([0,T]\times\mathbb{R}^{2n})$ is called a **viscosity subsolution** to the HJB equation

$$0 = -\frac{\partial v}{\partial t} + \sup_{u\in U}\left\{-\frac{\partial^\tau v}{\partial z}\psi\right\} - \frac{1}{2}\mathrm{Tr}\left(\frac{\partial^2 v}{\partial z^2}GG^\tau\right) - L$$
$$v|_{t=T} = h \qquad z \in \mathbb{R}^{2n} \tag{15}$$

if $\overline{v}|_{t=T} \le h$, and for any $\phi(t,z) \in C^{1,2}([0,T]\times\mathbb{R}^{2n})$, whenever $\overline{v} - \phi$ takes a local maximum at $(t,z) \in [0,T)\times\mathbb{R}^{2n}$, we have

$$-\frac{\partial \phi}{\partial t} + \sup_{u\in U}\left\{-\frac{\partial^\tau \phi}{\partial z}\psi\right\} - \frac{1}{2}\mathrm{Tr}\left(\frac{\partial^2 \phi}{\partial z^2}GG^\tau\right) - L \le 0,\ z \in \mathbb{R}^{2n} \tag{16}$$

at (t,z). $\overline{v}(t,z) \in C([0,T]\times\mathbb{R}^{2n})$ is called a **viscosity supersolution** to (15) if $\overline{v}|_{t=T} \ge h$, and in (16) we have an opposite inequality at (t,z), whenever $\overline{v} - \phi$ takes a local minimum at $(t,z) \in [0,T)\times\mathbb{R}^{2n}$. $\overline{v}(t,z)$ is called a **viscosity solution** if it is both a viscosity subsolution and a viscosity supersolution. □

We introduce the function class $\mathcal{G}$ such that each $v(t,x,p) \in \mathcal{G}$ satisfies

i) $v \in C([0,T]\times\mathbb{R}^{2n})$;
ii) there exist $C,\ k_1,\ k_2 > 0$ such that $|v| \le C[1 + \sum_{i=1}^n e^{k_1|x_i|} + \sum_{i=1}^n (|x_i|^{k_2} + |p_i|^{k_2})]$, where the constants $C,\ k_1,\ k_2$ depend on each v.

Theorem 3.4: [19], [45] The value function v is a viscosity solution to the HJB equation

$$0 = -\frac{\partial v}{\partial t} + \sup_{u\in U}\left\{-\frac{\partial^\tau v}{\partial z}\psi\right\} - \frac{1}{2}\mathrm{Tr}\left(\frac{\partial^2 v}{\partial z^2}GG^\tau\right) - L$$
$$v(T,x,p) = 0. \tag{17}$$

Moreover, there exists a unique viscosity solution to the (17) in the class $\mathcal{G}$. □

IV. Suboptimal Approximation of the Value Function

A. Perturbation of the HJB Equation

As is pointed out in Section III, in general we cannot prove the existence of a classical solution to the HJB equation (17) due to the lack of uniform parabolicity. Now, we modify (17) by adding a perturbing term $(1/2)\sum_{i=1}^n \varepsilon^2(\partial^2 v/\partial p_i^2)$ and formally carrying out the minimization to get

$$0 = \frac{\partial v^\varepsilon}{\partial t} + \frac{1}{2}\sum_{i=1}^n \sigma_i^2 \frac{\partial^2 v^\varepsilon}{\partial x_i^2} + \frac{1}{2}\sum_{i=1}^n \varepsilon^2 \frac{\partial^2 v^\varepsilon}{\partial p_i^2} - \sum_{i=1}^n \frac{\partial v^\varepsilon}{\partial x_i} a_i(x_i + b_i) - \sum_{i=1}^n \left|\frac{\partial v^\varepsilon}{\partial p_i}\right| + L(x,p) \tag{18}$$

where we use v^ε to indicate the dependence on $\varepsilon > 0$. We seek a classical solution v^ε in the class $\mathcal{F}$.

i) $v^\varepsilon \in C^{1,2}((0,T)\times\mathbb{R}^{2n}) \cap C([0,T]\times\mathbb{R}^{2n})$.
ii) $|v^\varepsilon| \le C(1 + |p|^{k_1} + e^{k_2|x|})$, where $C,\ k_1,\ k_2$ depend on v^ε.
iii) $v^\varepsilon(T,x,p) = 0$.

We will prove the existence of a solution to (18) in $\mathcal{F}$ by an approximation approach. First we fix $0 < \varepsilon < 1$. For a positive integer d, we introduce $h^d(x,p) = h^d(z)$ such that $h^d(z) = 1$ for $|z| \le d$, $h^d(z) = 0$ for $|z| \ge d+1$, and $|h^d_{z_i}| \le 2$. Write the auxiliary equation

$$0 = \frac{\partial v^d}{\partial t} + \frac{1}{2}\sum_{i=1}^n \sigma_i^2 \frac{\partial^2 v^d}{\partial x_i^2} + \frac{1}{2}\sum_{i=1}^n \varepsilon^2 \frac{\partial^2 v^d}{\partial p_i^2} - \sum_{i=1}^n \frac{\partial v^d}{\partial x_i} a_i(x_i + b_i) h^d(z) - \sum_{i=1}^n \left|\frac{\partial v^d}{\partial p_i}\right| h^d(z) + L(x,p)h^d(z)$$
$$v^d(T,x,p) = 0. \tag{19}$$

Theorem 4.1: Equation (18) has a unique solution v^ε in the class $\mathcal{F}$ for all $\varepsilon > 0$.

Proof: The existence of a classical solution can be proved in a way similar to [10, Th. VI6.2], and it can be shown first that (19) admits a classical solution v^d in the class $\mathcal{F}$. Fix any $d_0 > 1$. We take $D = (0,T) \times (|(x,p)| < d_0)$. Then, for any $d \geq d_0$, $v^d(t,x,p)$ in (19) satisfies (18) for $|z| < d_0$. v^d, $v^d_{x_i}$, $v^d_{p_i}$ are uniformly bounded on D, and for any $Q = (0,T) \times (|z| < d')$, $0 < d' < d_0$, by local estimates [10]

$$|v^d|^{(2)}_{\lambda,Q} \triangleq |v^d|_{\lambda,Q} + \left|v^d_s\right|_{\lambda,Q} + \sum_i \left|v^d_{z_i}\right|_{\lambda,Q} + \sum_{i,j} \left|v^d_{z_i z_j}\right|_{\lambda,Q}$$

is uniformly bounded with respect to d, where $|\cdot|_{\lambda,Q}$ denotes the $L^\lambda(Q)$ norm. In the above we can take $\lambda > n+2$, and therefore by the Hölder estimates [10], $v^d_{z_i}$ satisfies a uniform Hölder condition on Q. We can further use the Hölder estimates to show that $v^d_s, v^d_{z_i z_j}, d = d_0, d_0+1, d_0+2, \cdots$, satisfy a uniform Hölder condition on Q. Finally, we use the Arzela–Ascoli theorem [34] to take a subsequence $\{d_{k_m}, m \geq 1\}$ of $\{d_k \triangleq d_0 + k, k \geq 1\}$ such that $v^{d_{k_m}}, v^{d_{k_m}}_s, v^{d_{k_m}}_{z_i}, v^{d_{k_m}}_{z_i z_j}$ converge uniformly to $v^\varepsilon, v^\varepsilon_s, v^\varepsilon_{z_i}, v^\varepsilon_{z_i z_j}$ on Q, respectively, as $m \to \infty$, where v^ε satisfies (18) and is in the class $\mathcal{F}$. By the growth condition of v^ε, we can use Itö's formula to show that any $v^\varepsilon \in \mathcal{F}$ satisfying (18) is the value function to a related stochastic control system, and thus it is a unique solution to (18) in $\mathcal{F}$. □

Theorem 4.2: For $0 < \varepsilon < 1$, compact $B \subset \mathbb{R}^{2n}$, if v^ε is the solution to (18) in class $\mathcal{F}$, then $v^\varepsilon \to v$ uniformly on $[0,T] \times B$, as $\varepsilon \to 0$, where v is the value function of (13).

Proof: Suppose $\{w_i, \nu_i, 1 \leq i \leq n\}$ are mutually independent standard Wiener processes. Write

$$dp^\varepsilon_i = u_i dt + \varepsilon d\nu_i \qquad 1 \leq i \leq n. \tag{20}$$

Let $\mathcal{U}^{w,\nu}$ denote $\sigma(w_i, \nu_i)$-adapted controls satisfying $|u_i| \leq 1$, $1 \leq i \leq n$. It can be shown that the optimal cost of (13) does not change when $\mathcal{U}$ is replaced by $\mathcal{U}^{w,\nu}$. In fact, subject to the admissible control set $\mathcal{U}$ or $\mathcal{U}^{w,\nu}$ we can prove that the value function to the controlled system (13) is the viscosity solution to the same associated HJB equation and the viscosity solution is unique (see Theorem 3.4). Hence, in the following we always take controls from $\mathcal{U}^{w,\nu}$. Furthermore, $v^\varepsilon(t,x,p)$ is the value function to the stochastic control problem associated with (1) and (20), i.e.,

$$\begin{aligned} v^\varepsilon(s,x,p) &= \inf_{u \in \mathcal{U}^{w,\nu}} J^\varepsilon(s,x,p,u) \\ &= \inf_{u \in \mathcal{U}^{w,\nu}} E\left[\int_s^T L\left(x_t, p^\varepsilon_t\right) dt | x_s = x, p^\varepsilon_s = p\right]. \end{aligned}$$

For a fixed $u \in \mathcal{U}^{w,\nu}$, we have $P\{\lim_{\varepsilon \to 0} \sup_{0 \leq t \leq T} |p^\varepsilon_i - p_i| = 0\} = 1$, and using Lebesgue's dominated convergence theorem [34], we obtain

$$|J^\varepsilon(s,x,p,u) - J(s,x,p,u)| \to 0 \qquad \text{as } \varepsilon \to 0$$

and, therefore, $v^\varepsilon(s,x,p) \to v(s,x,p)$ as $\varepsilon \to 0$. It is easy to verify that v^ε is uniformly bounded on $[0,T] \times B$ for $0 < \varepsilon < 1$. Furthermore, by taking two different initial conditions (s,x,p) and (s',x',p') we can show that on $[0,T] \times B$, v^ε is equicontinuous with respect to $0 < \varepsilon < 1$. By the Arzela–Ascoli theorem, $v^\varepsilon \to v$ uniformly on $[0,T] \times B$, as $\varepsilon \to 0$. □

B. Interpretation of the Control Law

In the HJB equation (17), the value function is described by the formal use of its first and second order derivatives, and the equation is interpreted in a viscosity solution sense. The optimal control is not specified as a function of time and the state variable globally due to the nondifferentiable points of the value function.

After the perturbation of HJB equation, the associated suboptimal cost function is differentiable everywhere. Then, the suboptimal control law is constructed by the rule

$$u = \arg\min_{u \in U} \frac{\partial^\tau v^\varepsilon}{\partial z} \psi = -\mathrm{sgn}\left(\frac{\partial v^\varepsilon}{\partial p}\right) \tag{21}$$

which gives a bang–bang control. We note that the suboptimal control law (21) resembles the up/down power control algorithms in [35] where at each discrete time instant the power is increased or decreased by a fixed amount and the increment is determined by the current power, the observed random channel gain and a target SIR. However, our method here differs from [35] since the fading dynamics modeled by (1) are incorporated into the calculation of the control law (21). In a discrete time implementation, we assume the time axis is evenly sampled by a period of ΔT. At time $k\Delta T$, $k = 0,1,2,\ldots$, the ith user only needs to increase or decrease its power by ΔT in the case $(\partial v^\varepsilon / \partial p_i)|_{t=k\Delta T} < 0$ or $(\partial v^\varepsilon / \partial p_i)|_{t=k\Delta T} > 0$, respectively; if $(\partial v^\varepsilon / \partial p_i)|_{t=k\Delta T} = 0$, the increment for p_i is set as 0. The significance of the suboptimal control law is that it gives a very simple scheme (i.e., increase or decrease the power by a fixed amount or keep the same power level) for updating the power of users by requiring limited information exchange between the base station and the users (in the current technology, the base station sends the power adjustment command to the users based on its information on the operating status of each user), and thus reduces implementational complexity.

On the other hand, it is seen that each user uses centralized information, i.e., the current powers and attenuations of all the users, to determine its own power adjustment. In general, to implement the centralized control law requires more information exchange between the base station and the individual users than in the case of static channels [36], [37].

V. Numerical Implementation of the Suboptimal Control Law

From the analysis in Section IV, we can see that for a numerical implementation, we only need to choose a small positive constant $\varepsilon > 0$ and solve (18) and the suboptimal control is determined in a feedback control form. We consider the case of two users with i.i.d. dynamics

$$dx_i = -a(x_i + b)dt + \sigma dw_i, \qquad i = 1,2, \quad 0 \leq t \leq 1.$$

We take the time interval [0,1] and use a performance function $E\int_0^1 L(x_t, p_t)dt$ with

$$\begin{aligned} L = {} & \left[e^{x_1}p_1 - 0.4(e^{x_1}p_1 + e^{x_2}p_2 + 0.25)\right]^2 \\ & + \left[e^{x_2}p_2 - 0.4(e^{x_1}p_1 + e^{x_2}p_2 + 0.25)\right]^2 + \lambda(p_1 + p_2). \end{aligned}$$

A. Numerical Scheme

In order to compute the suboptimal control law, we need to solve the approximation equation

$$\begin{aligned} 0 = &\, v_t + \frac{1}{2}\sigma^2 \left(v_{x_1x_1} + v_{x_2x_2}\right) + \frac{1}{2}\varepsilon^2 \left(v_{p_1p_1} + v_{p_2p_2}\right) \\ &- a(x_1+b)v_{x_1} - a(x_2+b)v_{x_2} - |v_{p_1}| - |v_{p_2}| + L \\ &v(1,x,p) = 0. \end{aligned} \tag{22}$$

This equation is solved by a standard difference scheme [2] in a bounded region $S = \{(t,x,p) : 0 \le t \le 1, -4 \le x_1, x_2 \le 3, |p_1|, |p_2| \le 3\}$. An additional boundary condition is added such that $v(t,x,p)|_{\overline{\partial}} = 0$, where $\overline{\partial} = \partial S - \{(t,x,p), t = 0\}$. Let $\delta t, h > 0$ be the step sizes, and denote $z = (x_1, x_2, p_1, p_2)^\tau$, $e_i = (0, \cdots, 1, \cdots, 0)^\tau$ where 1 is the ith entry in the row. We discretize (22) to get the difference equation

$$\begin{aligned} 0 = &\frac{1}{\delta t}\left[v(t+\delta t, z) - v(t,z)\right] \\ &+ \frac{\sigma^2}{2h^2}\left[v(t, z+e_1h) + v(t, z-e_1h) - 2v(t,z)\right] \\ &+ \frac{\sigma^2}{2h^2}\left[v(t, z+e_2h) + v(t, z-e_2h) - 2v(t,z)\right] \\ &+ \frac{\varepsilon^2}{2h^2}\left[v(t, z+e_3h) + v(t, z-e_3h) - 2v(t,z)\right] \\ &+ \frac{\varepsilon^2}{2h^2}\left[v(t, z+e_4h) + v(t, z-e_4h) - 2v(t,z)\right] \\ &- \frac{a(x_1+b)}{h}\left[v(t, z+e_1h) - v(t,z)\right] 1_{\{a(x_1+b)\le 0\}} \\ &- \frac{a(x_1+b)}{h}\left[v(t,z) - v(t, z-e_1h)\right] 1_{\{a(x_1+b) > 0\}} \\ &- \frac{a(x_2+b)}{h}\left[v(t, z+e_2h) - v(t,z)\right] 1_{\{a(x_2+b)\le 0\}} \\ &- \frac{a(x_2+b)}{h}\left[v(t,z) - v(t, z-e_2h)\right] 1_{\{a(x_2+b) > 0\}} \\ &+ \frac{u_1}{2h}\left[v(t, z+e_3h) - v(t, z-e_3h)\right] \\ &+ \frac{u_2}{2h}\left[v(t, z+e_4h) - v(t, z-e_4h)\right] + L(z) \end{aligned} \tag{23}$$

where

$$u_i = -\mathrm{sgn}\left[v(t, z+e_{i+2}h) - v(t, z-e_{i+2}h)\right], \qquad i = 1,2. \tag{24}$$

With the boundary condition and an initial approximate solution, we can determine the variables u_1 and u_2 (the control variables) by the rule (24), and update the numerical solution. The iterations converge to the exact solution to the difference equation (23), as can be proved by the method in [26]. We remark that there are general results concerning the convergence of this type of difference scheme to the solution of the original partial differential equation. The interested reader is referred to the literature (see, e.g., [11]).

For a comparison, we also construct the power updating scheme for two users

$$\begin{aligned} Q_i(k\Delta T + \Delta T) = Q_i(k\Delta T) + \delta\, \mathrm{sgn}\left(\gamma_i - \Gamma_i(k\Delta T)\right) \\ k \ge 0, \qquad i = 1,2 \end{aligned} \tag{25}$$

where $\Gamma_1 = e^{x_1}p_1/(e^{x_2}p_2 + 0.25)$, $\Gamma_2 = e^{x_2}p_2/(e^{x_1}p_1 + 0.25)$. In the cost L for the simulation, $\mu_i = \gamma_i/(1+\gamma_i) = 0.4$, which determines $\gamma_1 = \gamma_2 = 2/3$. The rule (25) is used to mimic practical binary power control algorithms which are based on SIR targeting without taking into account the channel dynamics; see the discussion in [14].

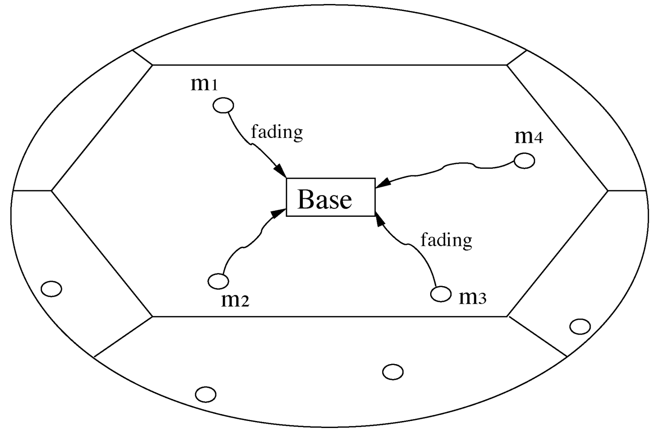

Fig. 1. Typical cell.

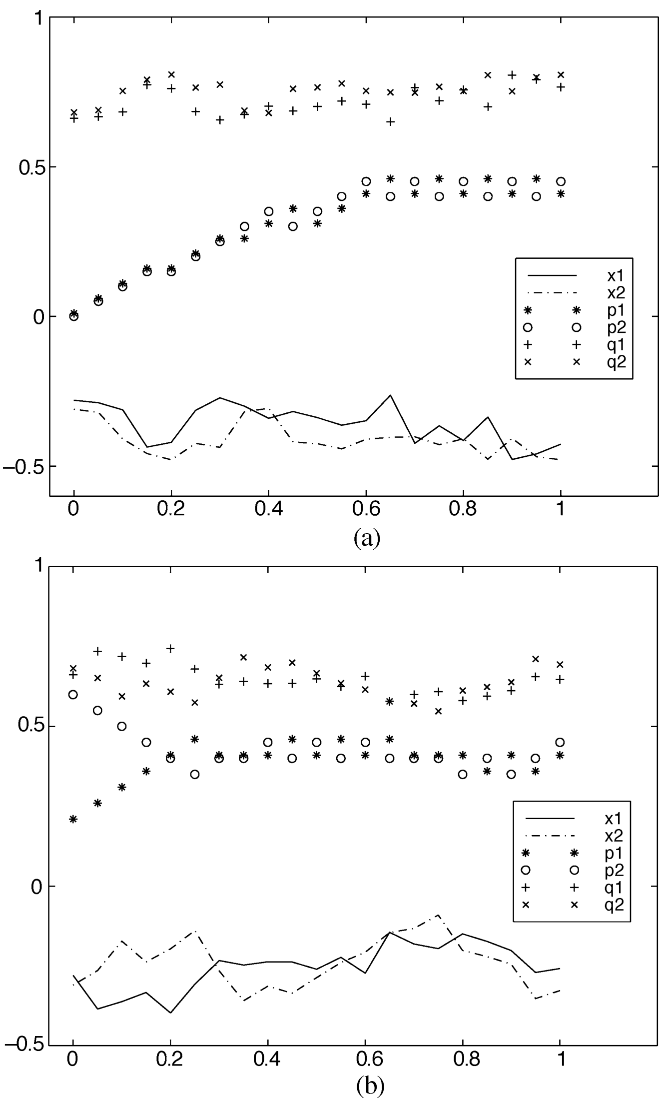

Fig. 2. Attenuation x_i, controlled power p_i, and static optimum q_i, $\lambda = 0.01$. Initial power: (a) $p_0 = [0.01, 0]^\tau$ and (b) $p_0 = [0.21, 0.6]^\tau$.

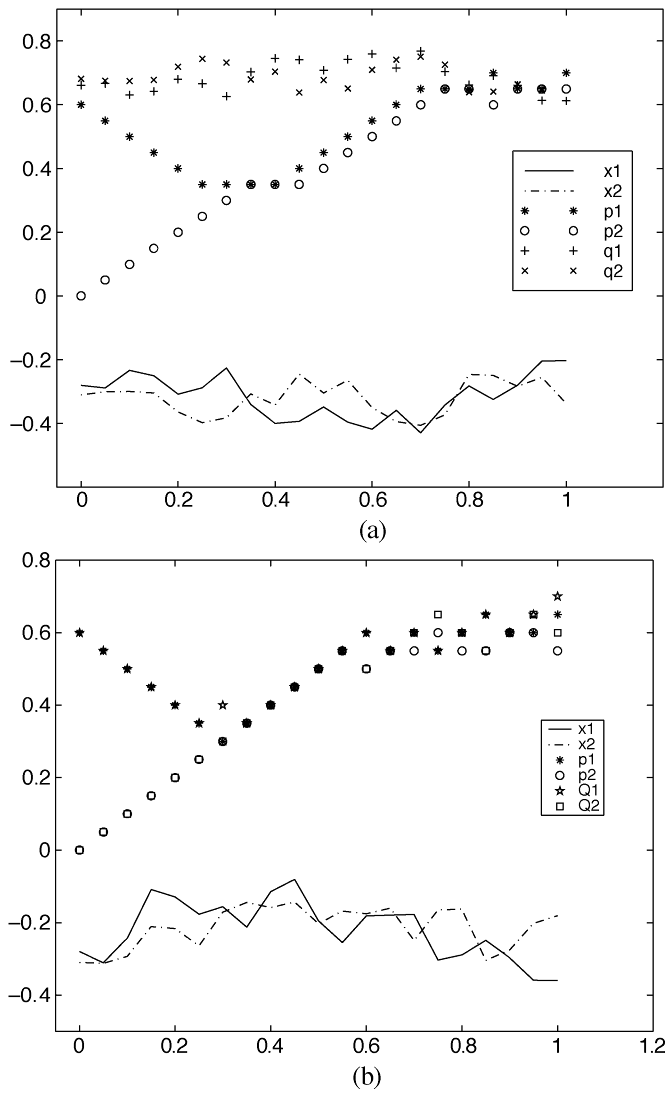

Fig. 3. (a) Attenuation x_i, controlled power p_i, static optimum q_i, $\lambda = 0$, $p_0 = [0.6, 0]^\tau$. (b) Comparison between p_i and Q_i determined by (25).

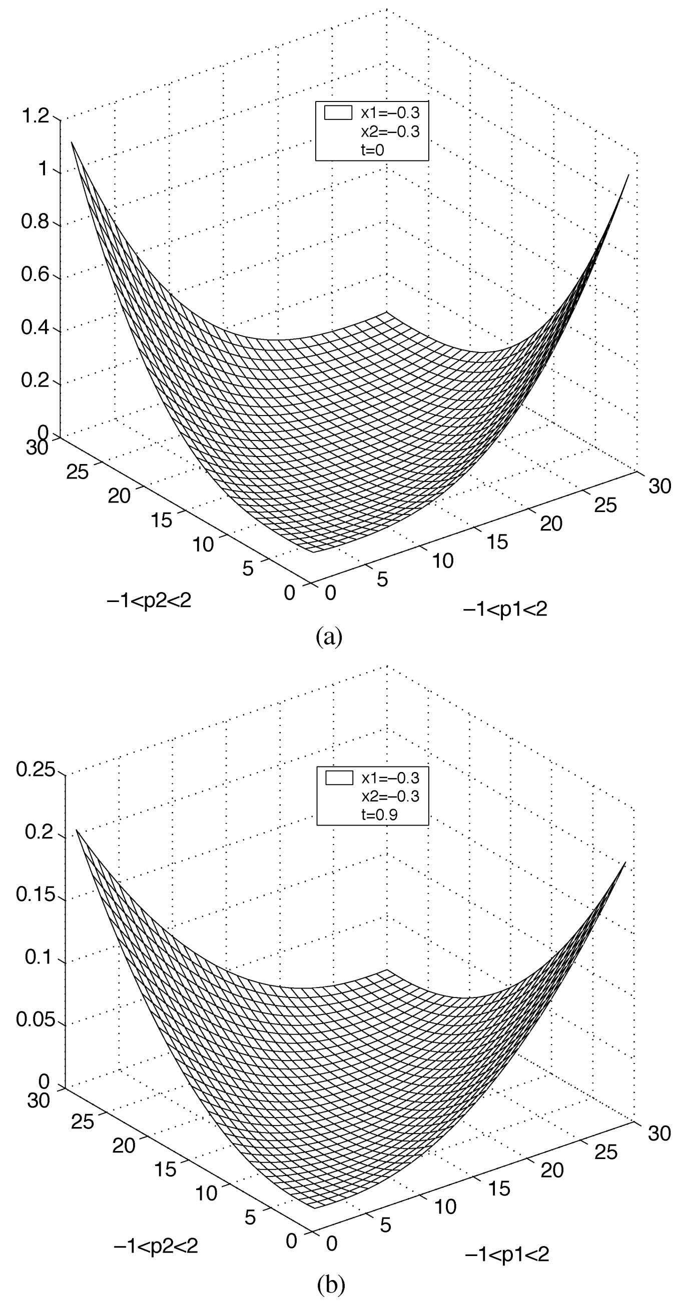

Fig. 4. Approximate surfaces for the value function $v(t, x_1, x_2, p_1, p_2)$ where $(x_1, x_2) = (-0.3, -0.3)$, $\lambda = 0$, (a) $t = 0$, and (b) $t = 0.9$.

B. Simulations

We consider the system with parameters $a = 4$, $b = 0.3$, $\sigma^2 = 0.09$, $\varepsilon^2 = 0.15$, and two cases for λ: 1) $\lambda = 0.01$; 2) $\lambda = 0$. In the difference scheme (23), the step size is 0.1 for t, x_i, p_i, $i = 1, 2$. To improve the approximation we can reduce ε, and at the same time we should reduce h to guarantee convergence of iterations of the difference scheme. In the simulation, the value function will be further interpolated to get a step size of 0.05 which will help reduce overshoot in the power adjustment. The control is determined by the descent direction of the value function. If either increasing or decreasing the power level does not cause an evident decrease of the value function, we set the control to be 0. Fig. 1 indicates the uplink power transmission from the users to the base station [33]. Figs. 2 and 3 present the simulation results for cases 1–2, respectively, and q_1, q_2 are the pointwise optimal powers (i.e., static optimum) obtained from (7). When the cost function places a small emphasis upon power saving $(\lambda = 0)$ the controlled power trajectories are seen to be close to the pointwise optimal powers. Fig. 4 shows two surfaces of the value function at different times when the attenuations are fixed. These surfaces clearly demonstrate the gradient information of the value function with respect to powers.

When at the initial time one mobile has a significantly different power level than the other, we see that an interesting equalization phenomenon takes place as shown in Figs. 2(b) and 3(a). Starting from the initial instant the controller will first make the mobile with a high power level reduce power and the other increase power; after a certain period however both mobiles will increase their power together. This phenomenon reveals a certain cooperative feature in the users' power adjustment. Suppose at the beginning user 1 has a high power and thus a high SIR output for itself; by slowly decreasing its power for a short period user 1 may still attain an acceptable SIR while effectively helping user 2 reach a desired SIR level in an accelerated manner. This feature is due to i.i.d. channel dynamics and the specific structure of the quadratic penalty function for the power vector. When the two powers are very different, they are necessarily far away from the minima of the quadratic form and incur a large instantaneous penalty. Then, an efficient way for the controller to work is to eliminate the large power difference by pushing two powers toward each other and eventually bring the two powers to steady levels.

We compare the trajectories of p_i and Q_i in Fig. 3(b). It turns out that during the equalizing phase the two corresponding control algorithms act in the same manner. This generates the fully overlapped segment of the trajectories (for instance, "box" and "circle"). However, an evident discrepancy is demonstrated between the two control laws (24) and (25) at the late stage. This takes place because the channel dynamics are employed in the calculation of the suboptimal control law and a prediction ability for the channel state is incorporated into the controller. Hence, the suboptimal control law (24) can react to the channel variation in a more clever manner than the rule (25).

VI. Discounted Case

A. Discounted Cost Function and the HJB Equation

In this section, we impose no bound constraint on the control input u and introduce a penalty term for u in the cost function. We write

$$E\int_0^\infty e^{-\rho t}\left\{\sum_{i=1}^n\left[e^{x_i}p_i-\mu_i\left(\sum_{j=1}^n e^{x_j}p_j+\eta\right)\right]^2+u^\tau Ru\right\}dt \tag{26}$$

where $\rho>0$ and $R>0$ is a weight matrix. We penalize abrupt change of powers via $u^\tau Ru$ since practical power control is exercised in a cautious manner and there exist basic limits for power adjustment rate. After subtracting the constant component from the integrand in (26), we get the cost function to be employed

$$J(u)=E\int_0^\infty e^{-\rho t}\left[p^\tau C(x)p+2D^\tau(x)p+u^\tau Ru\right]dt \tag{27}$$

where $C(x)$, $D(x)$ are $n\times n$ positive definite matrix, and $n\times 1$ vector, respectively, which are determined from (26). Write $l(x,p,u)=p^\tau C(x)p+2D^\tau(x)p+u^\tau Ru$.

We take the admissible control set $\mathcal{U}_2=\{u|u$ adapted to $\sigma(x_s,s\le t)$, and $E\int_0^\infty e^{-\rho t}|u_t|^2dt<\infty\}$. As in Section III, we can define the value function v. We do not repeat those here but will use the notation of Section III for which the interpretation should be clear. We note that certain controls from $\mathcal{U}_2$ may result in an infinite cost due to the presence of the e^{x_i} process, $1\le i\le n$. However the optimal control problem is still well defined under the admissible control set $\mathcal{U}_2$. We formally write the HJB equation for the value function v as

$$\begin{aligned}0=\rho v-f^\tau\frac{\partial v}{\partial x}-\frac{1}{2}\mathrm{Tr}\left(\frac{\partial^2 v}{\partial z^2}GG^\tau\right)\\
+\sup_{u\in\mathbb{R}^n}\left\{-u^\tau\frac{\partial v}{\partial p}-u^\tau Ru\right\}-p^\tau C(x)p-2D^\tau(x)p\end{aligned}$$

which gives

$$\begin{aligned}\rho v=-\sum_{i=1}^n a_i(x_i+b_i)\frac{\partial v}{\partial x_i}+\frac{1}{2}\sum_{i=1}^n\sigma_i^2\frac{\partial^2 v}{\partial x_i^2}\\
-\frac{1}{4}v_p^\tau R^{-1}v_p+p^\tau C(x)p+2D^\tau(x)p.\end{aligned} \tag{28}$$

Proposition 6.1: The value function v is a continuous function of (x,p) and can be written as

$$v(x,p)=p^\tau K(x)p+2p^\tau S(x)+q(x) \tag{29}$$

where $K(x)$, $S(x)$ and $q(x)$ are continuous in x, and are all of order $O(1+\sum_{i=1}^n e^{2x_i})$.

Proof: The continuity of v can be proved by continuous dependence of the cost on the initial condition of the system. Define $v_{n,N,T}(x,p)=\inf_{u\in\mathcal{U}_{2,T}^n}E[\int_0^T e^{-\rho t}l(x_t,p_t,u_t)1_{|x_t|\le N}dt|x_0=x,p_0=p]$ where $T>0$ and n, N are both positive integers; on $[0,T]$ the control set $\mathcal{U}_{2,T}^n\triangleq\{u|u$ adapted to $\sigma(x_s,s\le t)$ with stepwise value on $[iT/2^n,((i+1)T)/2^n],0\le i\le 2^n-1\}$. By a discrete-time LQ approach we solve $v_{n,N,T}(x,p)$ as a quadratic form in p. On the other hand, sending n, N, $T\to\infty$ in the sequel, it can be shown that $v_{n,N,T}(x,p)\to v(x,p)$ by an approximation argument [16]. Consequently, by convergence of $v_{n,N,T}$ we get the existence of $K(x)$, $S(x)$, $q(x)$ and the expression (29) for v. The upper bound for $K(x)$, $S(x)$, $q(x)$ is obtained by a direct estimate of the growth rate of v. □

Proposition 6.2: The value function v is a classical solution to the HJB equation (28), i.e., $\partial v/\partial x_i$, $\partial^2 v/\partial x_i^2$, $\partial v/\partial p_i$, $1\le i\le n$, exist and are continuous in $\mathbb{R}^{2n}$.

Sketch of Proof: By a vanishing viscosity technique [10], [43] one can show that the value function v is a generalized solution to (28) in terms of weak derivatives with respect to (x,p). By Proposition 6.1, we see that $\partial v/\partial p$ exists and is continuous. Now, (28) can be looked at as a partial differential equation parametrized by p. Then one can further show by use of smooth test functions of the form $\varphi_1(x)\varphi_2(p)$ with compact support that v is a generalized solution with respect to x for each fixed p. By a comparison method [10] using different initial values for x one can show that for each fixed p, $v(x,p)$ and hence $K(x)$, $S(x)$ all satisfies a local Lipschitz condition w.r.t. x. So for each fixed p, the term $\Psi^p(x)\triangleq-(1/4)v_p^\tau R^{-1}v_p+p^\tau C(x)p+2D^\tau(x)p$ in (28) also satisfies a local Lipschitz condition w.r.t. x. For a fixed p, (28) can be written in the form

$$-\rho v-\sum_{i=1}^n a_i(x_i+b_i)\frac{\partial v}{\partial x_i}+\frac{1}{2}\sum_{i=1}^n\sigma_i^2\frac{\partial^2 v}{\partial x_i^2}+\Psi^p(x)=0. \tag{30}$$

Since (28) is uniformly elliptic and Ψ^p is locally Lipschitz continuous w.r.t. x, the generalized solution v (w.r.t. x) has continuous first and second order derivatives with respect to x [12]. Hence v has all the classical derivatives appearing in the HJB equation (28). □

B. PDEs for the Discounted Case and the Control Law

By Propositions 6.1 and 6.2, we have

$$\begin{aligned} &p^\tau \rho K(x)p + 2p^\tau \rho S(x) + \rho q(x) \\ &\quad = f^\tau(x)\frac{\partial}{\partial x}\left[p^\tau K(x)p + 2p^\tau S(x) + q(x)\right] \\ &\quad\quad + \frac{1}{2}\mathrm{Tr}\left\{GG^\tau \frac{\partial^2}{\partial z^2}\left[p^\tau K(x)p + 2p^\tau S(x) + q(x)\right]\right\} \\ &\quad\quad - \left[K(x)p + S\right]^\tau R^{-1}\left[K(x)p + S\right] \\ &\quad\quad + p^\tau C(x)p + 2D^\tau(x)p. \end{aligned}$$

This gives

$$\begin{aligned} &p^\tau \rho K(x)p + 2p^\tau \rho S(x) + \rho q(x) \\ &\quad = p^\tau\left(\sum_{k=1}^n f_k \frac{\partial K}{\partial x_k}\right)p + 2p^\tau\left(\sum_{i=1}^n f_k \frac{\partial S}{\partial x_k}\right) + f^\tau \frac{\partial q}{\partial x} \\ &\quad\quad + p^\tau\left(\sum_{k=1}^n \frac{\sigma_k^2}{2}\frac{\partial^2 K}{\partial x_k^2}\right)p + p^\tau \sum_{k=1}^n \sigma_k^2 \frac{\partial^2 S}{\partial x_k^2} \\ &\quad\quad + \sum_{k=1}^n \frac{\sigma_k^2}{2}\frac{\partial^2 q}{\partial x_k^2} + p^\tau C p + 2D^\tau p - p^\tau K R^{-1} K p \\ &\quad\quad - S^\tau R^{-1} S - 2p^\tau K R^{-1} S. \end{aligned}$$

Hence, we get the partial differential equation system

$$\rho K = \frac{1}{2}\sum_{k=1}^n \sigma_k^2 \frac{\partial^2 K}{\partial x_k^2} + \sum_{k=1}^n f_k \frac{\partial K}{\partial x_k} - KR^{-1}K + C \quad (31)$$

$$\rho S = \frac{1}{2}\sum_{k=1}^n \sigma_k^2 \frac{\partial^2 S}{\partial x_k^2} + \sum_{k=1}^n f_k \frac{\partial S}{\partial x_k} - KR^{-1}S + D \quad (32)$$

$$\rho q = \frac{1}{2}\sum_{k=1}^n \sigma_k^2 \frac{\partial^2 q}{\partial x_k^2} + f^\tau \frac{\partial q}{\partial x} - S^\tau R^{-1} S \quad (33)$$

where we will refer to (31) as the Riccati equation of the system. Note that in the case the channel degenerates to a static one, i.e., $a_i = \sigma_i = 0$ for all i, (31) reduces to a usual algebraic Riccati equation. Finally, the optimal control law is given by

$$u = [u_1, \ldots, u_n]^\tau = -R^{-1}\left[K(x)p + S(x)\right] \quad (34)$$

where p denotes the power vector. This gives the control law for all users. The separation of variables x and p in (34) is useful, and this feature may be employed to construct simple suboptimal control laws as shown in Appendix B.

C. Simulations Based on the Discounted Cost Function

As in the bounded control case, we use a similar scheme to compute the value function approximately and the control law is determined by a quadratic type minimization based calculation. In the simulation, the system dynamics are the same as in Section V. In the quadratic type cost function, the discount factor $\rho = 0.5$, the weight matrix $R = 0.03I_2$, and $\mu = 0.4$, $\eta = 0.25$ as in Section V. The time step size is 0.05. In Fig. 5, x_i, p_i, u_i, $i = 1, 2$ denote the attenuation, controlled power, control, respectively, and Q_i is generated by the rule (25) in which we take $\delta = 0.02$, $\Delta T = 0.05$.

Similar to the bounded control case, the controlled powers also demonstrate a mutual convergence toward each other; however, after both powers settle down in a small neighborhood of a stable level, at each step only a minor effort is required for each mobile to adjust its power, which differs from the suboptimal bang–bang power control. The control activity is also much lower than the case for Q_i.

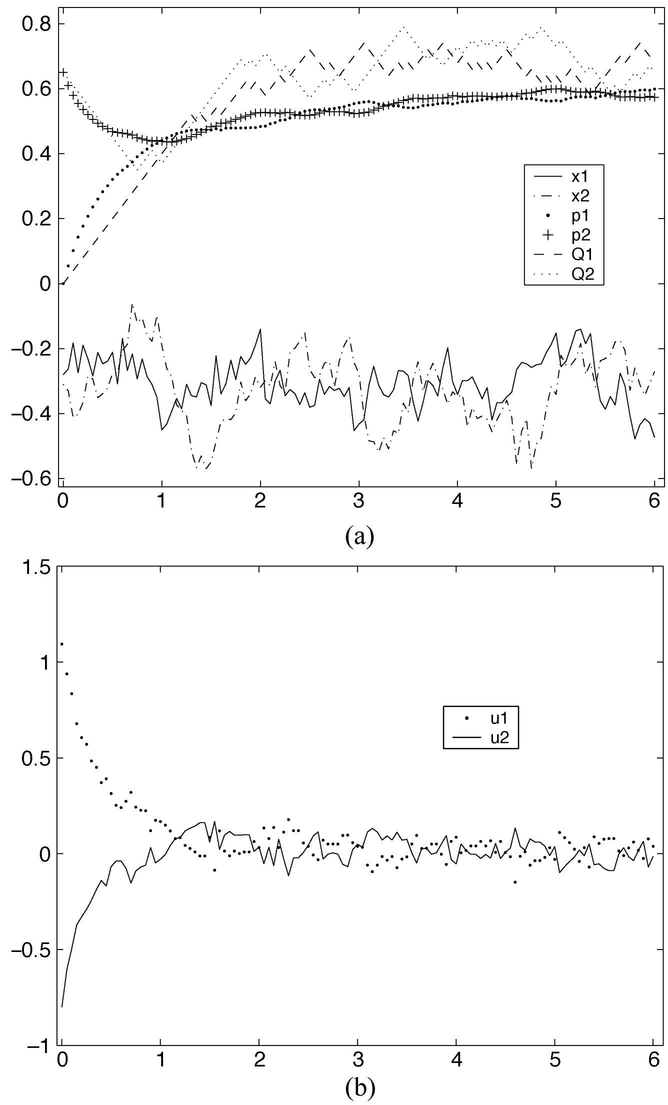

Fig. 5. (a) Attenuation x_i, controlled power p_i, and Q_i. (b) Control u_i.

D. Local Approximation of the Solution

In this section, we address the important issue of the computability of solutions to the equations in Section VI-B. An analysis of local expansions of solutions around a steady state mean $\overline{x}$ of the attenuation is useful in the small noise case because the attenuation trajectory $x(t)$ will be expected to spend a disproportionate amount of time in a small neighborhood of $\overline{x}$.

For simplicity, we consider the symmetric case, i.e., all the users have i.i.d. dynamics with $a_i = a$, $b_i = b$, $\sigma_i = \sigma$ and $\mu_i = \mu$, $R = rI_n$ in the cost. We use $K(x) = (K_{ij}(x))_{i,j=1}^n$ to denote the solution of the Riccati equation (31) and write

$$\begin{aligned} K_{ij}(x) &= K_{ij}(\overline{x}) + (x-\overline{x})^\tau K_{ij}^{(1)}(\overline{x}) \\ &\quad + \frac{1}{2}(x-\overline{x})^\tau K_{ij}^{(2)}(\overline{x})(x-\overline{x}) + o\left(|x-\overline{x}|^2\right) \quad (35) \end{aligned}$$

where $\overline{x} = (-b, \ldots, -b)^\tau$ and $K_{ij}^{(1)}(\overline{x}) = \partial K_{ij}(\overline{x})/\partial x$, $K_{ij}^{(2)}(\overline{x}) = \partial^2 K_{ij}(\overline{x})/\partial x^2$. It is worth mentioning that for the symmetric case, in the local polynomial expansion of $K(x)$ by (35) the number of distinct entries in the three coefficient matrices does not exceed 15 as the dimension of the system increases. This can be demonstrated by employing certain symmetry properties of the matrix $K(x)$ [16], [22].

We write the Riccati equation(31) in terms of its components to obtain

$$\rho K_{ij}(x) = \frac{\sigma^2}{2}\sum_k \frac{\partial^2 K_{ij}(x)}{\partial x_k^2} + \sum_k f_k \frac{\partial K_{ij}(x)}{\partial x_k} - \sum_k \frac{1}{r} K_{ik}(x) K_{kj}(x) + C_{ij}(x). \quad (36)$$

Now, we write the system of approximating equations (up to second order)

$$\begin{aligned}\rho K_{ij}(\overline{x}) &+ \rho(x-\overline{x})^\tau K_{ij}^{(1)}(\overline{x}) + \frac{\rho}{2}(x-\overline{x})^\tau K_{ij}^{(2)}(\overline{x})(x-\overline{x}) \\ &= \frac{\sigma^2}{2}\sum_k K_{ij,k}^{(2)}(\overline{x}) + \sum_k [-a(x_k - \overline{x}_k)] \\ &\quad \times \left[K_{ij,k}^{(1)}(\overline{x}) + K_{ij,k(\cdot)}^{(2)}(\overline{x})(x-\overline{x})\right] \\ &\quad - \sum_k \frac{1}{r} \times \Big[K_{ik}(\overline{x}) + (x-\overline{x})^\tau K_{ik}^{(1)}(\overline{x}) \\ &\qquad + \frac{1}{2}(x-\overline{x})^\tau K_{ik}^{(2)}(\overline{x})(x-\overline{x})\Big] \\ &\quad \times \Big[K_{kj}(\overline{x}) + (x-\overline{x})^\tau K_{kj}^{(1)}(\overline{x}) \\ &\qquad + \frac{1}{2}(x-\overline{x})^\tau K_{kj}^{(2)}(\overline{x})(x-\overline{x})\Big] + C_{ij}(\overline{x}) \\ &\quad + (x-\overline{x})^\tau C'_{ij}(\overline{x}) + \frac{1}{2}(x-\overline{x})^\tau C''_{ij}(\overline{x})(x-\overline{x}) \end{aligned} \quad (37)$$

where $K_{ij,k}^{(2)}(\overline{x})$, $K_{ij,k(\cdot)}^{(2)}(\overline{x})$ are the kth diagonal entry and the kth row of the matrix $K_{ij}^{(2)}(\overline{x})$, respectively, and $K_{ij,k}^{(1)}(\overline{x})$ is the kth entry of $K_{ij}^{(1)}(\overline{x})$. Notice that in writing (37) only the first three terms in (35) are formally substituted into (36) and the higher order terms are neglected. When the higher order terms are taken into account, additional terms of the order $|(\sigma^2/2)K_{ij}^{(3)}|$ and $|(\sigma^2/4)K_{ij}^{(4)}|$ will appear in (39) and (40), respectively, where $K_{ij}^{(3)}$ and $K_{ij}^{(4)}$ denote the third- and fourth-order mixing partial derivatives of $K_{ij}(x)$ at $\overline{x}$. Here, in order to avoid an infinitely coupled equation system we neglect these additional terms but maintain sufficiently close approximation to the exact solution since we are considering the small noise case. However, we write an exact equation corresponding to the zero-order term since it has more weight in the suboptimal control law when the state stays in a small neighborhood of $\overline{x}$. By grouping terms with zero power of $(x-\overline{x})$ in (37), we obtain the equation system

$$\rho K_{ij}(\overline{x}) = \frac{\sigma^2}{2}\sum_k K_{ij,k}^{(2)}(\overline{x}) - \frac{1}{r}\sum_k K_{ik}(\overline{x})K_{kj}(\overline{x}) + C_{ij}(\overline{x}) \quad (38)$$

or, equivalently, in the matrix form

$$\rho K(\overline{x}) = \frac{\sigma^2}{2}\left(\mathrm{Tr}\left\{K_{ij}^{(2)}(\overline{x})\right\}\right)_{i,j=1}^n - \frac{1}{r}K(\overline{x})K(\overline{x}) + C(\overline{x})$$

which takes the form of a perturbed algebraic Riccati equation. By (37), we also have

$$\begin{aligned}(x-\overline{x})^\tau \rho K_{ij}^{(1)}(\overline{x}) &= (x-\overline{x})^\tau \left(-aK_{ij,k}^{(1)}(\overline{x})\right)_{k=1}^n - \sum_k (x-\overline{x})^\tau \\ &\times \frac{1}{r}\left[K_{ik}^{(1)}(\overline{x})K_{kj}(\overline{x}) + K_{ik}(\overline{x})K_{kj}^{(1)}(\overline{x})\right] + (x-\overline{x})^\tau C'_{ij}(\overline{x})\end{aligned}$$

which gives

$$(\rho + a)K_{ij}^{(1)}(\overline{x}) = \frac{-1}{r}\sum_k \Big[K_{ik}^{(1)}(\overline{x})K_{kj}(\overline{x}) + K_{ik}(\overline{x})K_{kj}^{(1)}(\overline{x})\Big] + C'_{ij}(\overline{x}). \quad (39)$$

Finally, by inspecting the second-order terms in (37) we get

$$\left(\frac{\rho}{2}+a\right)K_{ij}^{(2)}(\overline{x}) = \frac{-1}{2r}\sum_k\Big[K_{ik}(\overline{x})K_{kj}^{(2)}(\overline{x}) + K_{kj}(\overline{x})K_{ik}^{(2)}(\overline{x})\Big] - \frac{1}{r}\sum_k K_{ik}^{(1)}(\overline{x})\Big[K_{kj}^{(1)}(\overline{x})\Big]^\tau + \frac{1}{2}C''_{ij}(\overline{x}). \quad (40)$$

It would be of interest to investigate the procedure to solve the equation system (38)–(40) numerically, which is an important step toward implementing the suboptimal control law in a simple and efficient manner. In Appendix B, a simple yet informative example of a single user is given to illustrate the interaction between the individual equations in the above system, and numerical methods can be devised to solve the equations of the example iteratively. However, we note that the analysis for the single user system carries special significance. In a system with many users, under reasonable conditions the (suitably scaled) interference which a given user receives (due to all other users and the background noise) can be approximated by a deterministic quantity (see also the analysis for SIR in large systems using the notion of effective interference in [9]); and, subsequently, any particular mobile user may be singled out for analysis. It turns out that the single user based control design can be effectively applied to systems with many users; see [22]. The single user based dynamic power control is also justified in [5] by a small variance assumption on the total received power at the based station.

VII. Conclusion

This paper initiates a stochastic control approach for uplink cellular power adjustment in the presence of lognormal fading communication channels for which a bounded rate power adjustment model is proposed. The existence of such a bound is implicit in current implementations [32] and it highlights the need to account for channel dynamics in developing optimal controls. Different cost functions have been introduced here which reflect the SIR requirements at the user level. In this framework, the control input involves information which is centralized through the base station. Numerical solutions in this paper to the two different formulations of the optimal control problem (with i.i.d. channel dynamics) reveal an initial equalization phase of the users' powers followed by motion toward a time varying optimal value. In addition, the paper presents an approximate, scalable solution to one of the optimal control problems. Furthermore, we have shown that adaptive control laws can be developed based upon online estimates of the channel parameters. The important issues of implementing the proposed control schemes in the case of large randomly varying network populations and the decentralization of the associated control laws will be investigated in future work (see, e.g., [20]).

Appendix A

Proof of Theorem 3.1

The existence of the optimal control can be established by a typical approximation argument and the details are omitted here (see, e.g., [43]).

Uniqueness: Assume there is $\widetilde{u} \in \mathcal{U}$ such that $J(x_0,p_0,\widetilde{u}) = J(x_0,p_0,\widehat{u})$, and denote the power corresponding to $\widetilde{u}$ by $\widetilde{p}$. Since for each fixed x, by Assumption H1) it can be verified that $\partial^2 L(x,p)/\partial p^2$ is strictly positive definite and, therefore, $L(x,p)$ is strictly convex with respect to p, we have

$$L\left(x_s, \frac{1}{2}(\widehat{p}_s+\widetilde{p}_s)\right) \le \frac{1}{2}\left[L(x_s,\widehat{p}_s)+L(x_s,\widetilde{p}_s)\right] \tag{A.1}$$

and a strict inequality holds on the set $A^0 \triangleq \{(s,\omega), \widetilde{p}_s \neq \widehat{p}_s\}$. Suppose $E\int_0^T 1_{(\widehat{p}_s\neq\widetilde{p}_s)}ds > 0$, i.e., A^0 has a strictly positive measure; then the control $(1/2)(\widehat{u}+\widetilde{u}) \in \mathcal{U}$ yields

$$\begin{aligned} J\left(x_0,p_0,\frac{1}{2}(\widehat{u}+\widetilde{u})\right) &< \frac{1}{2}\left[J(x_0,p_0,\widehat{u})+J(x_0,p_0,\widetilde{u})\right] \\ &= \inf_{u\in\mathcal{U}} J(x_0,p_0,u) \end{aligned}$$

by integrating and taking expectation on both sides of (A.1), which is a contradiction, and, therefore

$$E\int_0^T 1_{(\widehat{p}_s\neq\widetilde{p}_s)}ds = 0. \tag{A.2}$$

Since with probability 1 the trajectories of p_s are continuous, by (A.2) we have $\widetilde{p}_s - \widehat{p}_s \equiv 0$ on $[0,T]$ with probability 1. By (2) we have $\int_0^s(\widetilde{u}_t-\widehat{u}_t)dt = \widehat{p}_s-\widetilde{p}_s$, for all $s\in[0,T]$, so that with probability 1, $\widetilde{u}_s-\widehat{u}_s = 0$ a.e. on $[0,T]$ or, equivalently

$$E\int_0^T 1_{(\widetilde{u}_s\neq\widehat{u}_s)}ds = \int_0^T P_\Omega(\widetilde{u}_s\neq\widehat{u}_s)ds = 0.$$

So that $P_\Omega(\widetilde{u}_s\neq\widehat{u}_s) > 0$ only on a set of times $s\in[0,T]$ of Lebesgue measure zero. This proves uniqueness. □

Appendix B
Analysis of a Single User System

For illustrating the solution of the algebraic equation system in Section VI-D, we consider the simple example of $n=1$. This corresponds to the case of a single mobile user in service under the effect of a fading channel and the background noise. In this case, we have $C(\overline{x}) = (1-\mu)^2e^{-2b}$, $C'(\overline{x}) = 2(1-\mu)^2e^{-2b}$, $C''(\overline{x}) = 4(1-\mu)^2e^{-2b}$, and (38)–(40) reduce to

$$\rho K = \frac{\sigma^2}{2}K^{(2)} - \frac{1}{r}K^2 + C \tag{B.1}$$

$$(\rho+a)K^{(1)} = -\frac{2}{r}KK^{(1)} + C' \tag{B.2}$$

$$\left(\frac{\rho}{2}+a\right)K^{(2)} = -\frac{1}{r}KK^{(2)} - \frac{1}{r}K^{(1)}K^{(1)} + \frac{1}{2}C'' \tag{B.3}$$

where C, C' and C'' take their values at $\overline{x}$. In the following, we seek a solution for the small noise case satisfying $K \ge 0$.

Proposition B.1: There exists $\sigma_0^2 > 0$ such that for any $\sigma^2 \le \sigma_0^2$ the equation system (B.1)–(B.3) has a solution $(\overline{K}, \overline{K}^{(1)}, \overline{K}^{(2)})$ satisfying $\overline{K} \ge 0$.

Proof: Rewriting (B.1)–(B.3) yields

$$\begin{aligned} K &= \frac{-r\rho + \sqrt{r^2\rho^2 + 2r\left(\sigma^2K^{(2)} + 2C\right)}}{2} \\ &\triangleq G_0\left(K^{(2)}\right) \end{aligned} \tag{B.4}$$

$$K^{(1)} = \frac{rC'}{r(\rho+a)+2K} \triangleq G_1(K) \tag{B.5}$$

$$K^{(2)} = \frac{rC'' - 2K^{(1)}K^{(1)}}{r(\rho+2a)+2K} \triangleq G_2\left(K, K^{(1)}\right). \tag{B.6}$$

We now introduce four constants

$$\begin{aligned} c_1 &\triangleq \frac{C'}{\rho+a} \quad c_2 \triangleq \frac{C''}{\rho+2a} \\ c_2^- &\triangleq \inf_{0\le s\le G_0(c_2)} \frac{rC''-2c_1^2}{r(\rho+2a)+2s} \\ c_0 &\triangleq \frac{-r\rho + \sqrt{r^2\rho^2 + 2r(\sigma^2c_2 + 2C)}}{2} \end{aligned}$$

and a convex compact subset of $\mathbb{R}^3$ $\mathcal{K} \triangleq \{(x_0,x_1,x_2) : 0 \le x_i \le c_i, i=0,1 \text{ and } c_2^- \le x_2 \le c_2\}$. Set $\sigma_0^2 = \sup\{\sigma^2 : \sigma^2c_2^- + 2C \ge 0\}$. Then, for any $\sigma^2 \le \sigma_0^2$, the square root in (B.4) is always no less than $r\rho$ for $c_2^- \le K^{(2)} \le c_2$. We define the continuous map G on $\mathcal{K}$ such that

$$G\left(K, K^{(1)}, K^{(2)}\right) = \left(G_0\left(K^{(2)}\right), G_1(K), G_2\left(K, K^{(1)}\right)\right). \tag{B.7}$$

It is readily verified that $G(\mathcal{K}) \subseteq \mathcal{K}$ and, therefore, by Brouwer's fixed point theorem G has a fixed point $(\overline{K}, \overline{K}^{(1)}, \overline{K}^{(2)})$. From (B.4), it follows that $\overline{K} \ge 0$. Thus we have proved that the system (B.1)–(B.3) has a solution $(\overline{K}, \overline{K}^{(1)}, \overline{K}^{(2)})$ and $\overline{K} \ge 0$. □

We can further establish a contractive property for the map G under certain conditions by examining the Jacobian of $G(K, K^{(1)}, K^{(2)})$, and then the unique solution can be found by successive iterations of G. We proceed to consider the local approximation of $S(x)$ in Section VI-B. Write

$$S(x)=S(\overline{x})+S^{(1)}(\overline{x})(x-\overline{x})+\frac{1}{2}S^{(2)}(\overline{x})(x-\overline{x})^2+o\left(|x-\overline{x}|^2\right).$$

Similar to the treatment for $K(x)$, from (32), we obtain a system of algebraic equations

$$\begin{aligned}\left(\rho+\frac{\overline{K}}{r}\right)S-\frac{\sigma^2}{2}S^{(2)}&=D\\ \frac{\overline{K}^{(1)}}{r}S+\left(a+\rho+\frac{\overline{K}}{r}\right)S^{(1)}&=D'\\ \frac{\overline{K}^{(2)}}{2r}S+\frac{\overline{K}^{(1)}}{r}S^{(1)}+\left(\frac{\rho}{2}+a+\frac{\overline{K}}{2r}\right)S^{(2)}&=\frac{1}{2}D''\end{aligned}$$

where $D=D'=D''=-\mu\eta(1-\mu)e^{-b}$.

Example 1: For $n=1$, $a=2$, $b=0.3$, $\sigma^2=0.01$, $\mu=0.6$, $\eta=0.25$, $\rho=0.5$, and $r=0.1$, we have

$$\begin{aligned}\left(\overline{K},\overline{K}^{(1)},\overline{K}^{(2)}\right)&=(0.072\,12,\ 0.044\,55,\ 0.052\,43)\\ \left(\overline{S},\overline{S}^{(1)},\overline{S}^{(2)}\right)&=(-0.036\,41,\ -0.008\,763,\ -0.003\,362).\end{aligned}$$

Example 2: For $n=1$, $a=2$, $b=0.4$, $\sigma^2=0.01$, $\mu=0.6$, $\eta=0.25$, $\rho=0.5$, and $r=0.1$, we have

$$\begin{aligned}\left(\overline{K},\overline{K}^{(1)},\overline{K}^{(2)}\right)&=(0.063\,52,\ 0.038\,13,\ 0.044\,79)\\ \left(\overline{S},\overline{S}^{(1)},\overline{S}^{(2)}\right)&=(-0.035\,44,\ -0.008\,517,\ -0.003\,475).\end{aligned}$$

Remark: For $x=(x_1,x_2,x_3)\in\mathbb{R}^3$, define $\|x\|=\max_i|x_i|$. By examining the upper bounds for $|\partial G_j/\partial x_i|$ on $\mathcal{K}$, $j=0,1,2$, $i=1,2,3$, where G is defined by (B.7), we can show that in Examples 1–2, the map G is a contraction on $\mathcal{K}$ under $\|\cdot\|$. □

The suboptimal control law for the single user is determined by substituting the local polynomial approximation of $K(x)$ and $S(x)$ into the feedback control given in Section VI-B, i.e.,

$$\begin{aligned}u=&-\frac{1}{r}\left[\overline{K}+\overline{K}^{(1)}(x+b)+\frac{1}{2}\overline{K}^{(2)}(x+b)^2\right]p\\ &-\frac{1}{r}\left[\overline{S}+\overline{S}^{(1)}(x+b)+\frac{1}{2}\overline{S}^{(2)}(x+b)^2\right].\qquad\text{(B.8)}\end{aligned}$$

By retaining only the constant terms in (B.8), we get the zero-order approximation of the optimal control law as $u^{(0)}=-(\overline{K}/r)p-(\overline{S}/r)$, for which the steady state power is $p^\infty=-(\overline{S}/\overline{K})$. On the other hand, we determine the nominal power level by setting $e^{\overline{x}}\overline{p}-\mu(e^{\overline{x}}\overline{p}+\eta)=0$. Define the relative error between p^∞ and $\overline{p}$ by $Err(p^\infty,\overline{p})\triangleq|p^\infty-\overline{p}|/\overline{p}$. For Examples 1 and 2, a comparison is listed in Table I.

Fig. 6 demonstrates the dynamic behavior of the system in Example 1 under the suboptimal control law (B.8).

The single-user-based analysis can be useful when applied to systems with large populations. In that case a particular user views other user interference as background noise. This leads to a partially decentralized and effective power control scheme [16].

TABLE I
COMPARISON BETWEEN p^∞ AND $\overline{p}$

Example	p^∞	$\overline{p}$	$Err(p^\infty,\overline{p})$
1	0.504881	0.506197	$< 0.3\%$
2	0.557938	0.559434	$< 0.3\%$

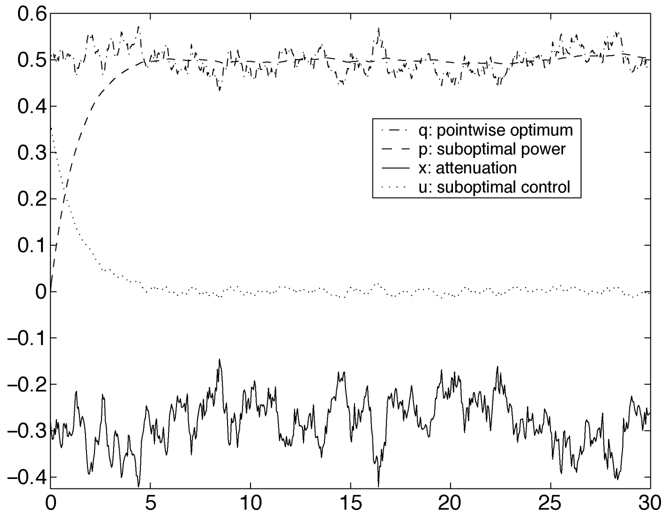

Fig. 6. Single user controller with initial power $p_0=0$.

APPENDIX C
ADAPTATION WITH UNKNOWN CHANNEL PARAMETERS

We rewrite the lognormal fading channel model of Section II-A as follows:

$$dx_i=-a_i(x_i+b_i)dt+\sigma_i dw_i,\qquad 1\le i\le n.\qquad\text{(C.1)}$$

In this model, the channel variation is characterized by the parameters $a_i>0$, $b_i>0$, $\sigma_i>0$. For practical implementations, a_i, b_i, σ_i may not be known a priori, but x_i can be measured, for instance, with the aid of pilot signals [31]. In CDMA systems, the power of users is updated with a period close to 1 millisecond (for instance, by 800 Hz [39]) while the time scale of lognormal fading is much larger. Hence, the channel may be regarded as varying at a very slow rate. In such a case, one expects to have estimation of the channel state with high accuracy. In the following analysis, we will assume perfect knowledge on the channel state x_i.

Consider an estimation algorithm for a_i and b_i via the measurement of x_i. For the ith mobile, the parameters are estimated by the least squares algorithm where $\widehat{a}_i(t)$, $\widehat{b}_i(t)$ denote the estimate of a_i, b_i at $t\ge 0$, respectively. Define

$$\widehat{b}_i(t)=-\frac{1}{t}\int_0^t x_i(s)ds,\qquad t>0\qquad\text{(C.2)}$$

$$\begin{aligned}dP_i=&-P_i(x_i+\widehat{b}_i)(x_i+\widehat{b}_i)P_idt,\qquad t\ge 0\qquad\text{(C.3)}\\ d\widehat{a}_i=&-P_i(x_i+\widehat{b}_i)\left[dx_i+\widehat{a}_i(x_i+\widehat{b}_i)dt\right],\\ &t\ge 0\qquad\text{(C.4)}\end{aligned}$$

where the initial conditions are given by $\widehat{b}_i(0)$, $P_i(0) > 0$, $\widehat{a}_i(0)$, respectively. The algorithm (C.3)–(C.4) may be regarded as a modified Kalman filtering algorithm for constant parameters with random observations; a discrete time version of this algorithm was first proposed in [28]. The resulting estimates are strongly consistent as stated by the following proposition.

Proposition C.1: The estimates $\widehat{b}_i(t)$ and $\widehat{a}_i(t)$ converge to the true parameters with probability one as $t \to \infty$, i.e.,

$$\lim_{t\to\infty} \widehat{b}_i(t) = b_i, \qquad \text{a.s.} \tag{C.5}$$
$$\lim_{t\to\infty} \widehat{a}_i(t) = a_i, \qquad \text{a.s.} \tag{C.6}$$

with initial conditions $\widehat{b}_i(0)$, $\widehat{a}_i(0)$, $P_i(0) > 0$.

Proof: Since $a_i > 0$, $\sigma_i > 0$, it follows that x_i is an ergodic diffusion process satisfying

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t x_i(s)ds = \lim_{t\to\infty} Ex_i(t) = -b_i, \qquad \text{a.s.}$$

and (C.5) follows. Write $\widetilde{a}_i = \widehat{a}_i - a_i$, $\widetilde{b}_i = \widehat{b}_i - b_i$ and $y_i = x_i + \widehat{b}_i$. It is easy to verify that

$$\begin{aligned} d\widetilde{a}_i &= -P_i(x_i + \widehat{b}_i)\left[dx_i + \widehat{a}_i(x_i + \widehat{b}_i)dt\right] \\ &= -P_i\widetilde{a}_i y_i^2 dt - a_i\widetilde{b}_i P_i y_i dt - \sigma_i P_i y_i dw_i \qquad \text{(C.7)} \\ dP_i^{-1} &= y_i^2 dt. \qquad \text{(C.8)} \end{aligned}$$

By (C.8), it follows that

$$\begin{aligned} &\left|\frac{1}{t}P_i^{-1}(t) - \frac{1}{t}\int_0^t (x_i+b_i)^2 ds\right| \\ &= \left|\frac{1}{t}P_i^{-1}(0) + \frac{1}{t}\int_0^t (b_i - \widehat{b}_i)^2 ds - \frac{2}{t}\int_0^t (x_i+b_i)(b_i-\widehat{b}_i)ds\right| \\ &\le \frac{1}{t}P_i^{-1}(0) + \frac{1}{t}\int_0^t \widetilde{b}_i^2 ds + 2\left(\frac{1}{t}\int_0^t (x_i+b_i)^2 ds\right)^{\frac{1}{2}} \\ &\quad \times \left(\frac{1}{t}\int_0^t \widetilde{b}_i^2 ds\right)^{\frac{1}{2}}. \end{aligned} \tag{C.9}$$

Since $\lim_{t\to\infty}(1/t)\int_0^t (x_i + b_i)^2 ds = \lim_{t\to\infty} E[x_i(t) - Ex_i(t)]^2$ a.s. by ergodicity of x_i, and $\lim_{t\to\infty}(1/t)\int_0^t \widetilde{b}_i^2 ds = 0$ a.s., it follows that

$$\lim_{t\to\infty} \frac{1}{t}P_i^{-1}(t) = \lim_{t\to\infty} E\left[x_i(t) - Ex_i(t)\right]^2 > 0, \qquad \text{a.s.} \tag{C.10}$$

By (C.7) and (C.8), we obtain

$$d\left(P_i^{-1}\widetilde{a}_i^2\right) = -\widetilde{a}_i^2 y_i^2 dt - 2a_i\widetilde{b}_i\widetilde{a}_i y_i dt + \sigma_i^2 P_i y_i^2 dt - 2\sigma_i\widetilde{a}_i y_i dw_i. \tag{C.11}$$

Applying the technique in [4], from (C.11) we get

$$\begin{aligned} &P_i^{-1}(t)\widetilde{a}_i^2(t) - P_i^{-1}(0)\widetilde{a}_i^2(0) \\ &\quad = -\int_0^t \widetilde{a}_i^2 y_i^2 ds + \int_0^t \sigma_i^2 P_i y_i^2 dt - 2\int_0^t a_i\widetilde{b}_i\widetilde{a}_i y_i ds \\ &\qquad - 2\int_0^t \sigma_i\widetilde{a}_i y_i dw_i \\ &\quad = -\int_0^t \widetilde{a}_i^2 y_i^2 ds + O\big(\log P_i^{-1}(t)\big) \\ &\qquad + O\left(\left(\int_0^t \widetilde{a}_i^2 y_i^2 ds\right)^{\frac{1}{2}+\varepsilon}\right) \\ &\qquad + O\left(\left[\int_0^t \widetilde{b}_i^2 ds\right]^{\frac{1}{2}} \cdot \left[\int_0^t \widetilde{a}_i^2 y_i^2 ds\right]^{\frac{1}{2}}\right), \qquad \text{a.s.} \end{aligned} \tag{C.12}$$

where $0 < \varepsilon < (1/2)$. From (C.12), it follows that

$$\begin{aligned} \int_0^t \widetilde{a}_i^2 y_i^2 ds \le O\left(\log P_i^{-1}(t)\right) + O\left(\left(\int_0^t \widetilde{a}_i^2 y_i^2 ds\right)^{\frac{1}{2}+\varepsilon}\right) \\ + O\left(\left[\int_0^t \widetilde{b}_i^2 ds\right]^{\frac{1}{2}} \cdot \left[\int_0^t \widetilde{a}_i^2 y_i^2 ds\right]^{\frac{1}{2}}\right) \end{aligned}$$

which yields

$$\int_0^t \widetilde{a}_i^2 y_i^2 ds = O(\log t) + O\left(\int_0^t \widetilde{b}_i^2 ds\right), \qquad \text{a.s.}$$

Since $\widetilde{b}_i(t) \to 0$, a.s., as $t \to \infty$, it follows that

$$\frac{1}{t}\int_0^t \widetilde{a}_i^2 y_i^2 ds \to 0, \qquad \text{a.s.} \tag{C.13}$$

By (C.10), (C.12), and (C.13), we get $\lim_{t\to\infty} \widetilde{a}_i(t) = 0$, a.s., and (C.6) follows. □

In the following, we employ a discrete time prediction error term to construct the empirical variance. We first take a sampling step $h > 0$ to discretize (C.1) as

$$\begin{aligned} x_i\left[(k+1)h\right] + b_i &= e^{-a_i h}\left[x_i(kh) + b_i\right] \\ &\quad + \sigma_i \int_{kh}^{(k+1)h} e^{-a_i[(k+1)h-s]} dw_i(s), \qquad k \ge 0. \end{aligned} \tag{C.14}$$

Setting $\nu_i(kh) = \int_{kh}^{(k+1)h} e^{-a_i[(k+1)h-s]}dw_i(s)$ and $A_i = e^{-a_i h}$, (C.14) can be written in the form

$$x_i[(k+1)h] + b_i = A_i[x_i(kh) + b_i] + \sigma_i \nu_i(kh).$$

It is easy to verify that $\mathrm{Var}(\nu_i(kh)) = (1 - e^{-2a_i h})/(2a_i) \triangleq \Sigma_{\nu_i}$. Denote $\widehat{A}_i(kh) = e^{-\widehat{a}_i(kh)h}$, $\widehat{\Sigma}_{\nu_i}(kh) = (1 - e^{-2\widehat{a}_i(kh)h})/(2\widehat{a}_i(kh))$ and

$$\widehat{\sigma}_i^2(nh) = \frac{1}{n\widehat{\Sigma}_{\nu_i}(kn)} \sum_{k=0}^{n-1} \Big(x_i[(k+1)h] + \widehat{b}_i(kh) - \widehat{A}_i(kh)\Big[x_i(kh) + \widehat{b}_i(kh)\Big]\Big)^2. \quad \text{(C.15)}$$

It is straightforward to show that (C.15) can be written in a recursive form.

Proposition C.2: For $\widehat{\sigma}_i^2(nh)$, $n \geq 1$, defined by (C.15), we have

$$\lim_{n\to\infty} \widehat{\sigma}_i^2(nh) = \sigma_i^2, \qquad \text{a.s.} \quad \text{(C.16)}$$

where $\sigma_i^2 > 0$ is determined by (C.1).

Proof: For notational brevity, in the proof we write $x_i(kh)$, A_i, b_i, $\widehat{A}_i(kh)$, $\widehat{b}_i(kh)$, $\nu_i(kh)$ as $x_i(k)$, A, b, $\widehat{A}(k)$, $\widehat{b}(k)$, $\nu(k)$, respectively. Setting $\widetilde{A}(k) = \widehat{A}_i(kh) - A_i$ and $\widetilde{b}(k) = \widehat{b}_i(kh) - b_i$, we have

$$\begin{aligned}
&\sum_{k=0}^{n-1} \Big(x_i[(k+1)h] + \widehat{b}_i(kh) - \widehat{A}_i(kh)\Big[x_i(kh) + \widehat{b}_i(kh)\Big]\Big)^2 \\
&= \sum_{k=0}^{n-1} \Big[\widetilde{A}(k)x_i(k)\Big]^2 + \sum_{k=0}^{n-1}\Big[Ab - \widehat{A}(k)\widehat{b}(k) - \widetilde{b}(k)\Big]^2 \\
&\quad + \sum_{k=0}^{n-1} \sigma_i^2 \nu^2(k) + 2\sum_{k=0}^{n-1}\Big[\widetilde{A}(k)x_i(k)\Big] \\
&\quad \times \Big[\widehat{A}(k)\widehat{b}(k) + \widetilde{b}(k) - Ab\Big] \\
&\quad + 2\sum_{k=0}^{n-1}\Big[-\widetilde{A}(k)x_i(k)\Big][\sigma_i \nu(k)] \\
&\quad + 2\sum_{k=0}^{n-1}\Big[Ab - \widehat{A}(k)\widehat{b}(k) - \widetilde{b}(k)\Big][\sigma_i\nu(k)] \\
&\triangleq S_1 + S_2 + S_3 + S_{12} + S_{13} + S_{23}. \qquad \text{(C.17)}
\end{aligned}$$

Since $\widehat{A}(k) \to A$, $\widehat{b}(k) \to b$ a.s., as $k \to \infty$, and $\sum_{k=0}^{n-1} x_i^2(k) = O(n)$ a.s., it follows that

$$|S_1| + |S_2| + |S_{12}| = o(n), \qquad \text{a.s.} \quad \text{(C.18)}$$

On the other hand, we have $S_{13} = O(S_1^{(1/2)+\varepsilon})$, $S_{23} = O(S_2^{(1/2)+\varepsilon})$ a.s. for any $0 < \varepsilon < (1/2)$ (see, e.g., [7]) and, therefore

$$|S_{13}| + |S_{23}| = o(n), \qquad \text{a.s.} \quad \text{(C.19)}$$

By (C.17)–(C.19), it follows that

$$\lim_{n\to\infty} \widehat{\sigma}_i^2(nh) = \lim_{n\to\infty} \frac{1}{n\widehat{\Sigma}_{\nu_i}(kn)} \sum_{k=0}^{n-1} \sigma_i^2\nu^2(k) = \sigma_i^2, \quad \text{a.s.}$$

which completes the proof. □

REFERENCES

[1] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," in *Proc. 40th IEEE Conf. Decision Control*, Orlando, FL, Dec. 2001, pp. 197–202.

[2] W. F. Ames, *Numerical Methods for Partial Differential Equations*, 3rd ed. New York: Academic, 1992.

[3] R. Buche and H. J. Kushner, "Control of mobile communications with time-varying channels in heavy traffic," *IEEE Trans. Automat. Contr.*, vol. 47, pp. 992–1003, June 2002.

[4] P. E. Caines, "Continuous time stochastic adaptive control: nonexplosion, ε-consistency and stability," *Syst. Control Lett.*, vol. 19, no. 3, pp. 169–176, 1992.

[5] J.-F. Chamberland and V. V. Veeravalli, "Decentralized dynamic power control for cellular CDMA systems," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 549–559, May 2003.

[6] C. D. Charalambous and N. Menemenlis, "Stochastic models for long-term multipath fading channels," in *Proc. 38th IEEE Conf. Decision Control*, Phoenix, AZ, Dec. 1999, pp. 4947–4952.

[7] H.-F. Chen and L. Guo, *Identification and Stochastic Adaptive Control*. Boston, MA: Birkhäuser, 1991.

[8] A. J. Coulson, G. Williamson, and R. G. Vaughan, "A statistical basis for lognormal shadowing effects in multipath fading channels," *IEEE Trans. Commun.*, vol. 46, pp. 494–502, Apr. 1998.

[9] J. Evans and D. N. C. Tse, "Large system performance of linear multiuser receivers in multipath fading channels," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2059–2078, Sept. 2000.

[10] W. H. Fleming and R. W. Rishel, *Deterministic and Stochastic Optimal Control*. Berlin, Germany: Springer-Verlag, 1975.

[11] W. H. Fleming and H. M. Soner, *Controlled Markov Processes and Viscosity Solutions*. New York: Springer-Verlag, 1993.

[12] A. Friedman, *Partial Differential Equations of Parabolic Type*. Upper Saddle River, NJ: Prentice-Hall, 1964.

[13] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electron. Lett.*, vol. 27, no. 23, pp. 2145–2146, 1991.

[14] S. V. Hanly and D. N. Tse, "Power control and capacity of spread spectrum wireless networks," *Automatica*, vol. 35, pp. 1987–2012, 1999.

[15] M. L. Honig and H. V. Poor, "Adaptive interference suppression," in *Wireless Communications: Signal Processing Perspective*, H. V. Poor and G. W. Wornell, Eds. Upper Saddle River, NJ: Prentice-Hall, 1998, pp. 64–128.

[16] M. Huang, "Stochastic control for distributed systems with applications to wireless communications," Ph.D. dissertation, Dept. Electr. Comput. Eng., McGill University, Montreal, QC, Canada, June 2003.

[17] M. Huang, P. E. Caines, C. D. Charalambous, and R. P. Malhamé, "Power control in wireless systems: a stochastic control formulation," in *Proc. Amer. Control Conf.*, Arlington, VA, June 2001, pp. 750–755.

[18] ——, "Stochastic power control for wireless systems: Classical and viscosity solutions," in *Proc. IEEE Conf. Decision Control*, Orlando, FL, Dec. 2001, pp. 1037–1042.

[19] M. Huang, P. E. Caines, and R. P. Malhamé, "Degenerate stochastic control problems with exponential costs and weakly coupled dynamics: viscosity solutions and a maximum principle," *SIAM J. Control Optim.*, 2004, submitted for publication.

[20] ——, "Distributed stochastic control for large-scale wireless networks in a game theoretic approach," McGill Univ., Montreal, QC, Canada, Intern. Rep., 2004.

[21] ——, "Individual and mass behavior in large population stochastic wireless power control problems: centralized and Nash equilibrium solutions," in *Proc. 42nd IEEE Conf. Decision Control*, Maui, HI, Dec. 2003, pp. 98–103.

[22] M. Huang, R. P. Malhamé, and P. E. Caines, "Stochastic power control in wireless communication systems with an infinite horizon discounted cost," in *Proc. Amer. Control Conf.*, Denver, CO, June 2003, pp. 963–968.

[23] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[24] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 46–55, Jan. 2002.

[25] O. E. Kelly, J. Lai, N. B. Mandayam, A. T. Ogielski, J. Panchal, and R. D. Yates, "Scalable parallel simulations of wireless networks with WIPPEP: Modeling of radio propagation, mobility and protocols," *Mobile Networks Applicat.*, vol. 5, no. 3, pp. 199–208, 2000.

[26] H. J. Kushner and A. J. Kleinman, "Numerical methods for the solution of the degenerated nonlinear elliptic equations arising in optimal stochastic control theory," *IEEE Trans. Automat. Contr.*, vol. AC-13, pp. 344–353, Aug. 1968.

[27] N. B. Mandayam, P. C. Chen, and J. M. Holtzman, "Minimum duration outage for cellular systems: A level crossing analysis," in *Proc. 46th IEEE Conf. Vehicular Technol.*, Atlanta, GA, Apr. 1996, pp. 879–883.
[28] D. Q. Mayne, "Parameter estimation," *Automatica*, vol. 3, pp. 245–255, 1966.
[29] D. Mitra and J. A. Morrison, "A novel distributed power control algorithm for classes of services in cellular CDMA networks," in *Advances in Wireless Communications*, J. M. Holtzman and M. Zorzi, Eds. Boston, MA: Kluwer, 1998, pp. 187–202.
[30] K. Pahlavan and A. H. Levesque, *Wireless Information Networks*. New York: Wiley, 1995.
[31] W. G. Phoel and M. L. Honig, "Performance of coded DS-CDMA with pilot-assisted channel estimation and linear interference suppression," *IEEE Trans. Commun.*, vol. 50, pp. 822–832, May 2002.
[32] QUALCOMM, Inc., "An overview of the application of code division multiple access (cdma) to digital cellular systems and personal cellular networks,", Document no. EX60-10 010, 1992.
[33] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 2002.
[34] W. Rudin, *Real and Complex Analysis*, 3rd ed. New York: McGraw-Hill, 1987.
[35] L. Song, N. B. Mandayam, and Z. Gajic, "Analysis of an up/down power control algorithm for the CDMA reverse link under fading," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 277–286, Feb. 2001.
[36] C. W. Sung and W. S. Wong, "A distributed fixed-step power control algorithm with quantization and active link quality protection," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 553–562, Mar. 1999.
[37] ——, "Mathematical aspects of the power control problem in mobile communication systems," in *Lectures on Systems, Control, and Information*, L. Guo and S. S.-T. Yau, Eds. Providence, RI: AMS/IP, 2000, vol. 17.
[38] ——, "Performance of a cooperative algorithm for power control in cellular systems with a time-varying link gain matrix," *Wireless Networks*, vol. 6, no. 6, pp. 429–439, 2000.
[39] "Mobile station-base station compatibility standard for dual-mode wideband spread spectrum cellular system," Telecommun. Industry Association, Tech. Rep. TIA/EIA/IS-95-A, 1995.
[40] H. Viswanathan, "Capacity of Markov channels with receiver CSI and delayed feedback," *IEEE Trans. Inform. Theory*, vol. 45, pp. 761–771, Mar. 1999.
[41] A. M. Viterbi and A. J. Viterbi, "Erlang capacity of a power-controlled CDMA system," *IEEE J. Select. Areas Commun.*, vol. 11, pp. 892–900, Sept. 1993.
[42] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341–1347, Nov. 1995.
[43] J. Yong and X. Y. Zhou, *Stochastic Controls: Hamiltonian Systems and HJB Equations*. New York: Springer-Verlag, 1999.
[44] J. Zhang, E. K. P. Chong, and I. Kontoyiannis, "Unified spatial diversity combining and power allocation for CDMA systems in multiple time-scale fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 1276–1288, July 2001.
[45] M. Huang, P. E. Caines, and R. P. Malhamé, "On a class of singular stochastic control problems arising in communications and thier viscosity solutions," in *Proc. IEEE Conf. Decision Control*, Orlando, FL, Dec. 2001, pp. 1031–1036.

Minyi Huang (S'01–M'04) received the B.Sc. degree from Shandong University, Shandong, China, in 1995, the M.Sc. degree from the Institute of Systems Science, Chinese Academy of Sciences, Beijing, China, in 1998, and the Ph.D. degree from the Department of Electrical and Computer Engineering, McGill University, Montreal, QC, Canada, in 2003, all in the area of systems and control.

He is now a Research Fellow with the Department of Electrical and Electronic Engineering, University of Melbourne, Victoria, Australia. His research interests include stochastic control and its application, power control in wireless networks, large-scale complex systems, communication rate constrained control systems, and sensor networks.

Peter E. Caines (M'74–SM'83–F'86) received the B.A. degree in mathematics from Oxford University, Oxford, U.K., in 1967, and the D.I.C. and Ph.D. degrees in systems and control theory from the University of London (Imperial College), London, U.K., in 1970.

Since 1980, he has been with McGill University, Montreal, QC, Canada, where he is a James McGill Professor and Macdonald Professor in the Department of Electrical and Computer Engineering. He was elected to the Royal Society of Canada in 2003, and is the author of *Linear Stochastic Systems* (New York: Wiley, 1988). His research interests lie in the areas of stochastic systems and hierarchical, hybrid, and discrete-event systems.

Roland P. Malhamé (S'82–M'92) received the B.Sc. degree from the American University of Beirut, the M.Sc. degree from the University of Houston, Houston, TX, and the Ph.D. degree from Georgia Institute of Technology, Atlanta, all in electrical engineering, in 1976, 1978, and 1983, respectively.

After single year stays at the University of Québec, Canada, and CAE Electronics Ltd. Montreal, QC, Canada, he joined the École Polytechnique de Montréal in 1985, where he is currently a Professor of electrical engineering. In 1994, he spent a sabbatical period at LSS, CNRS, France. His current research interests are in stochastic control and the analysis and optimization of complex networks, in particular, manufacturing and communication networks. His past contributions include a statistical mechanics-based approach to electric load modeling.