(2011.02)

Errata for Dixon and Mortimer "PERMUTATION GROUPS" (Springer 1996)

Chapter 1

11:-10 read "on each of its orbits of length > 1,"

13:13-15 read "Suppose that G is a group acting primitively on a set Ω and that Δ is a proper subset of Ω containing at least two points. Show that for each pair of distinct points ..."

13:18 add "[Hint: Show that the relation $\alpha \approx \beta \iff$ (for all $x \in G$, $\{\alpha, \beta\} \cap \Delta^x = \{\alpha, \beta\}$ or $\emptyset$) is a G-congruence.]"

17:-3 read "If $\Delta, \Delta' \in \Sigma$ are fixed setwise by H, then "

19:21 read "If $fix(G_\alpha)$ is finite, show it is a block for G."

22:2 read "... $= (\beta^{\sigma(a)})^{\sigma(x)} = (\lambda(\alpha^{\rho(a)}))^{\sigma(x)} = \lambda(\gamma)^{\sigma(x)}$ "

22:-13 read "Thus Lemma 1.6B shows ..."

23:9 read "and let $\alpha \in \Omega$. "

23:-2 *replace this incorrrect exercise by*

1.6.19 If x, y are distinct elements of order 2 in a finite group G, show that $\langle xy \rangle \lhd \langle x, y \rangle$ and hence that $\langle x, y \rangle$ is a dihedral group. Hence show that every primitive subgroup of order $2n$ in S_n is dihedral.

27:-10 read "acting transitively on a set Ω"

Chapter 2

30:14 read "–at least in principle–"

34:10 read "Suppose that G is a permutation group of degree at least 5. If G is k-transitive for some $k \geq 3$, show that every nontrivial normal subgroup N of G is $(k-2)$-transitive. In particular, ..."

35:12 read "(see Exercise 2.1.7)"

35:20 read "Hence show that S_n acts ..."

46:-9 read "when $x^{-1} =$"

48:-5 read "S_{p^m} ($\cong Sym(\Delta^m)$) "

51:12-14 read "a constant function in $Fun(\Gamma, \Delta)$ whose value lies in Π cannot be mapped under W to a constant function whose value lies in $\Delta \setminus \Pi$; thus W is intransitive. In the case ..."

51:-9 read "Define $g \in Fun(\Gamma, K)$"

51:-8 read "$[f(\gamma_0), u) \in K \setminus K_\delta$"

51:-4 read "$(1, x)B(\gamma_0)(1, x)^{-1} = B(\gamma_0^x)$

53:15 read "and $G_{\infty 0}$ transitive on the nonzero elements of F."

60:7 read "(1234), (13)"

60:14 read "(12)(34)(56), (153)(246)"

63:-16 read "$PGL_2(5) \cong S_5$"

Chapter 3

66:–9 read "the *diagonal* orbit $\Delta_1 := \{(\alpha, \alpha) \mid \alpha \in \Omega\}$; the other orbitals are called *nondiagonal*."

68:9 read "$H := \langle t \rangle$"

70:-16 delete "that G is finite,"
75:-19 read "A_3 is a composition factor"
84:-1 and 85:1,2 replace "2-cycle" by "3-cycle" and "$p \neq 2$" by "$p \neq 3$"
93:-8 read "$(a_1 + ... + a_k)^p = a_1^p + ... + a_k^p$ "
102:20 read "$w \in W$"

Chapter 4

109:-5 read "Show that $C \cong C_0 \, wr_\Sigma \, Sym(\Sigma)$ where ..."
110:4 read "each point stabilizer of H is its own normalizer in H, "
113:-2 read "p-group of order p^n"
114:20 read "$K \times C_G(K)$"
119:-10 read "with a finite nontrivial suborbit whose paired suborbit is also finite, show that "

{David Evans, Suborbits in infinite primitive permutation groups, *Bull. London Math. Soc.* **33** (2001) 583–590 gives a construction of an infinite primitive permutation group of arbitrary infinite cardinality with a finite nontrivial suborbit whose paired suborbit is infinite.}

124:-4 read "H is a transitive normal subgroup"

Chapter 5

163:4 read "5.5.2 Using the fact that $\lambda(s+1) \geq (2s-4)/3$ ..."
170:1-4 read "5.7.3 Show that A_6 is isomorphic to $SL_2(9)$ modulo its centre. Hence $\lambda(6) = 2$."
170:5-6 read "5.7.4 Show that there is no field F for which $SL_2(F)$ contains a finite preimage G of A_7. (However, A_7 is isomorphic to a section of $SL_3(25)$, and so $\lambda(7) = 3$.)"
170:13 read "For all $k \geq 5$, $\lambda(k) \geq (2k-6)/3$."
172:8 read "... Since $k \geq 8$, we have $d \geq 3$"
172:21-22 read "... shows that $d - 2 \geq \{2(k-3) - 6\}/3$ and hence $d \geq (2k-6)/3$ as required. ..."
172:after the last line add the following paragraph:

"Note that if $d = 3$ then the Jordan form for x cannot consist of a single block. Indeed, the centralizer of such a block is a group of upper triangular matrices and hence solvable, but we know that $C_G(x)$ is not solvable."

173:2 delete "(since $d \geq 4$)"

Chapter 6

181:3-5 The off-diagonal entries of AA^T should be λ_2. Then read: "The determinant of the $v \times v$ matrix AA^T is $(r + (v-1)\lambda_2)(r - \lambda_2)^{v-1}$ (see Exercise 6.2.2 below). This determinant is nonzero since $r > \lambda_2$ by the formulae above and our general assumption that $k < v$."
183:-6 read "and that $\mu_{ij} = \mu_{i-1,j} - \mu_{i,j+1}$"
207:2 read "Lemma 6.8B"

Chapter 7

210:-2 read "the stabilizers $G_{\alpha_1\alpha_2...\alpha_k}$ of k points"
217:10 read "Theorem 7.2C shows"
217:13 read "finite Frobenius"

239: Table 7.1 for each of the seven groups the generator a should read $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

244:-18 read "If $n > 8$, show"

248:-14 read "$G = Sp_4(2) \cong S_6$ and $H = G$."

251:19 read "σ^2 is the Frobenius automorphism $\xi \mapsto \xi^3$"

251:24 read "$\lambda_3 = \eta_1\eta_3^\sigma - \eta_1^{\sigma+1}\eta_2^\sigma + \eta_1^{\sigma+3}\eta_2 +$ "

251:-3 read "$(\eta_1, \eta_2, \eta_3, \lambda_1, \lambda_2, \lambda_3) \leftrightarrow (\lambda_2/\lambda_3, \lambda_1/\lambda_3, \eta_3/\lambda_3, \eta_2/\lambda_3, \eta_1/\lambda_3, 1/\lambda_3)$

{The permutation representation of $R(q)$ on p. 251 can be deduced, for example, from [KLM] G. Kemper, F. Luebeck and K. Magaard, "Matrix generators for the Ree groups ${}^2G_2(q)$", *Comm. Algebra* **29** (2001) 407-413 where the authors give explicit 7×7 matrices over $GF(q)$ generating $R(q)$. The 2-transitive permutation action of degree $q^3 + 1$ comes from right multiplication by $R(q)$ on the set of right cosets of the subgroup H consisting of all lower traingular matrices. If we define Q as the Sylow 3-subgroup consisting of the matrices $x_S(t, u, v)$ in [KLM], and use w to denote the involution denoted by n in [KLM], then $Q \cup \{w\}$ is a set of coset representatives of H. Using the parametrization $(\eta_1, \eta_2, \eta_3) = (t^\theta, -u^\theta, v^\theta - u^\theta t^\theta)$ for the coset with representative $x_S(t, u, v)$, and ∞ for Hw, we obtain the permutation representation on page 251 with $f_1 = \lambda_1$, $f_2 = \lambda_2$ and $f_3 = \lambda_3$.). Note that θ in [KLM] is the reciprocal of our σ.}

Chapter 8

256:15 read "has order $|\Omega|$ for $\mathbf{c} = \aleph_0$ and order at most $|\Omega|^{\mathbf{c}}$ for $\aleph_0 < \mathbf{c} \leq |\Omega|$."

262:12 read "Theorem 3.3C shows "

263: *replace the second paragraph by:*

Let $G \leq FSym(\Omega)$ be residually finite. We have to show that every orbit of G is finite. Suppose the contrary and let Σ be the union of the infinite G-orbits. Put $K := G_{(\Omega \setminus \Sigma)}$.

First note that if $H \leq G$ has finite index in G, then Σ is a union of infinite H-orbits. Indeed, if $\gamma \in \Sigma$, then $\left|\gamma^H\right| = |H : H_\gamma| \geq |G : G_\gamma| \,/\, |G : H|$.

We next show that K must be transitive on each infinite G-orbit Γ. Fix $\alpha, \beta \in \Gamma$ with $\alpha \neq \beta$ and choose $x \in G$ such that $\alpha^x = \beta$; we must show that $\alpha^z = \beta$ for some $z \in K$. Put $\Delta := \mathrm{supp}(x) \cap \Sigma$ and $\Phi := \mathrm{supp}(x) \setminus \Delta$. Since each point in the finite set Φ lies in a finite G-orbit, $G_{(\Phi)}$ has finite index in G, and so all the $G_{(\Phi)}$-orbits in Σ are infinite. Thus Theorem 3.3C shows that there exists $y \in G_{(\Phi)}$ such that the finite subset $\Delta \subseteq \Sigma$ satisfies $\Delta^y \cap \Delta = \emptyset$. Since the supports of x and y on the invariant subset $\Omega \setminus \Sigma$ are disjoint, $z := xyx^{-1}y^{-1}$ leaves all points in $\Omega \setminus \Sigma$ fixed, and so z lies in K. On the other hand, $\beta^y \in \Delta^y \subseteq \Sigma \setminus \Delta$ and so $\beta^y \notin \mathrm{supp}(x)$. Therefore $\alpha^z = (\beta^y)^{x^{-1}y^{-1}} = \beta$ as required. This proves the transitivity of K on each infinite G-orbit.

Finally, note that for each subgroup H of finite index in K, Lemma 8.3C(i) shows that $(K^\Sigma)' \leq H^\Sigma$ and so $K' \leq H$. Since K is a subgroup of a residually finite group G, K is also residually finite, and so the intersection of all subgroups of finite index in K must be 1. Thus $K' = 1$ and so K is abelian. However, if Γ is an infinite K-orbit, then Lemma 8.3C(ii) applied to K^Γ shows that $Z(K^\Gamma) = 1$. Thus $K^\Gamma = 1$ contradicting the transitivity of K on Γ. This completes the proof.

Remark 1 *This proof is based on P.M. Neumann, "The structure of finitary permutation groups", Archiv Math. 27 (1976) 3-17.*

Appendix B *(These corrections are due to Heiko Theissen and Colva Roney-Dougal)*

In Table B.2 the ranks of the normalizers of the following groups should be corrected:

A_9 (degree 840): rank 9; $L_2(5^2)$ (degree 325): rank 10; $L_3(2^2).3$ (degree 960): rank 10; $L_3(2^2).2$ (degree 336): rank 6; $U_3(2^2)$ (degree 208): rank 4 and (degree 416): rank 5; $S_4(2^2).4$ (degree 425): rank 5; $Sz(2^3)$ (degree 560): rank 7; M_{12} (degree 495): both of rank 8.

Also the normalizer for $H = L_2(p^2)$ (degree $p^2 + 1$ with p prime) should be $H.2^2$ and for $H = S_4(2^3)$ (degree 585) should be $H.3$.

In Table C.2 the normalizer for $H = L_3(3)$ (degree 13) should be H and for $L_3(4)$ (degree 21) should be $H.S_3$. Also under $L_3(q)$ (degree $q^2 + q + 1$) the lower bound should be 11 (see below).

In Table B.4 the following counts should be corrected:

Degree 91: there is only one cohort of type C ($L_3(9)$ is incorrectly listed twice)

Degree 244: there is only one cohort of type B

Degree 585: there is only one cohort of type E

Degree 364: there is a cohort of type C

Degree 384: there is no cohort of type C.

In Tables B.2 and B.4 for degree 574 there is a cohort missing for the group $L_2(41)$. It has stabilizer A_5, rank 16 and is its own normalizer in S_{574} (Colva Roney-Dougal 2004.06)

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