Homework 1
MATH4805/COMP4805/MATH5605 Theory of Automata
Fall 2008 – Due on October 23rd

(1) Let $A = \{0, 1\}$. Determine a regular expression for each of the following languages:

(i) $\{w \in A^* | w$ has at least two 0's$\}$

(ii) $\{w \in A^* | w$ has at exactly two 0's$\}$

(iii) $\{w \in A^* | w$ has an even number of 1's or an odd number of 0's$\}$

(iv) $\{w \in A^* | w$ starts with a 0 and ends with two 1's$\}$

(2) Construct a DFA recognizing each of the languages from question 1.

(3) Prove that if L_1, L_2 and L_3 are languages over A, then $L_1(L_2L_3) = (L_1L_2)L_3$.

(4) Let $A = \{., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let L be the language of all "correct decimal representations". That is, those strings which

(i) w cannot begin with a decimal point

(ii) w cannot end with a decimal point

(iii) w can have at most one decimal point

(iv) If the first symbol of w is 0, then the second symbol of w must be a decimal point

(v) If the last symbol of w is 0 then w does not have a decimal point.

For example, $23.456, 0.156 \in L$ and $3.12.8, .334, 0020, 000.45, 12., 3.60 \notin L$.

Construct a NFA that recognizes L.

(5) Construct a NFA recognizing

$$\{a^n | n \leq 4 \text{ or } n \equiv 3, 4 \bmod 5\}.$$

Using the *subset construction*, give a DFA that recognizes L.

(6) Prove that if $u, v \in A^*$, $|u| = |v|$ and $uv = vu$, then $u = v$.

(7) let $n \geq 1$. Show that the language $(0 + 1)^*1(0 + 1)^{n-1}$ can be recognized by an NFA with at most $n+1$ states. Give and justify a lower bound on the number of states a DFA must have to recognize this language. Find the best lower bound you can. Not all lower bounds will get full marks.

(8) Construct an ϵ-automata to recognize $(a^2b^* + b^2a^*)(ab + ba)$.

(9) Prove that the following equalities hold for regular expressions:

(i) $r^* = (rr) * + r(rr)^*$

(ii) $(r + s)^* = (r^*s^*)^*$

(iii) $(rs)^*r = r(sr)^*$

(10) Give the regular expression for the language recognized by the following automaton. Show all your work!!!!!!

δ	t	c	v
$\leftarrow H$	B	H	H
$\rightarrow B$	A	B	H
$\leftarrow A$	A	A	B

(11) Give an example of a language which is NOT recognizable but that DOES satisfy the conclusions of the pumping lemma. Do NOT give the example from the text and DO justify all your claims.