Homework 2
MATH4805/COMP4805/MATH5605 Theory of Automata
Fall 2008 – Due on November 27th

(1) Let $L = \{a^i b^j c^k | i > j > k\}$. Prove that L is not context-free.

(2) Find a PDA recognizing the language generated from the following grammar:

$$\begin{aligned} S &\rightarrow aTb|bTa|TT \\ T &\rightarrow bTb|aTa|\epsilon \end{aligned}$$

(3) Give a context-free grammar for the language

$$\{w \in \{a,b\}^* | w \text{ has even length and its middle two symbols are "}ba\text{"}\}.$$

(4) Give a PDA and a context-free grammar recognizing the languages $L_1 = \{a^m b^n c^n | m, n \geq 0\}$ and $L_2 = \{a^i b^j | i \geq j\}$.

(5) L_1 and L_2 are context-free languages. Prove or Disprove each of the following:

(i) $L_1 \cap L_2$ is context-free

(ii) $L_1 + L_2$ is context-free

(iii) $L_1 L_2$ is context-free

(iv) L_1^* is context-free

(v) L_1^c is context-free.

(6) If $u, v \in A^*$, we say that w is a *shuffle* of u and v if there exist factorizations $u = u_1 u_2 \cdots u_n$, $v = v_1 v_2 \cdots v_n$ (empty factors are allowed) such that $w = u_1 v_1 u_2 v_2 \cdots u_n v_n$. We write $u\#v$ for the set of all shuffles of u and v.

Use the Pumping Lemma to prove that the language

$$L = \{w\#x | x \in \{0,1\}^* \text{ and } w \text{ is a substring of } x\}$$

is not a context-free language.

(7) Minimize the following DFA:

	a	b
$\rightarrow s_0$	s_1	s_2
s_1	s_2	s_3
s_2	s_2	s_3
$\leftarrow s_3$	s_3	s_4
s_4	s_2	s_3

(8) Find the Minimal automaton recognizing $L = 1(0^*1)^*1 + \epsilon$. For this language, give a monoid, M, a homomorphism

$$f : \{0,1\}^* \to M$$

and a set $F \subseteq M$ such that $L = f^{-1}(F)$.

(9) Let G be a finite graph with vertices V and edges E. Let

$$L = \{w \in E^* | w = e_1 e_2 \cdots e_n \text{ is a valid walk in } G\}.$$

If we allow the empty walk as valid, show that L is regular

(10) The language $L = (0+1)^*1(0+1)^2$ can be recognized by a 4 state NFA and an 8 state DFA. Give the Transition monoids of each. The transition monoid of the DFA will be a submonoid of a row monomial monoid. The transition monoid of the NFA will be a submonoid of the $n \times n$ matrices over the Boolean algebra. What is the unique (up to isomorphism) minimal monoid that recognizes this language?