Exercises: Space Curves and Surfaces-2

Exercise Set 1

Find the equation of the tangent plane and the normal line to the surfaces defined by the following vector functions:

1. $z = 2x^2 - 4y^2 \quad$ at $(2, 1, 4)$.

2. $z = yx$ at $(1, 1, 1)$.

3. $x^3 - 3xy + y^3$ at $(1, 1, -1)$.

4. $z = \arctan\left(\frac{y}{x}\right)$ at $(1, 1, \frac{\pi}{4})$.

5. $z = \sqrt{x^2 + y^2} - yx$ at $(3, 4 - 7)$.

6. $x^3 + y^3 + z^3 + xyz - 6 = 0$ at $(1, 2, -1)$.

7. $(z^2 - x^2)xyz - y^5 - 5 = 0$ at $(1, 1, 2)$..

8. $4 - x - y - z + \sqrt{x^2 + y^2 + z^2} = 0$ at $(2, 3, 6)$.

The next two problems require more thought ...

9. Find the points on the surface
$$x^2 + y^2 + z^2 - 6y + 4z - 12 = 0$$
at which the tangent planes are parallel to one of the coordinate planes ($x = 0$, $y = 0$, $z = 0$).

10. Find the tangent plane to the surface
$$x^2 - y^2 = 3z$$
passing through $(0, 0, -1)$ and parallel to the line whose symmetric equations are
$$\frac{x}{2} = y = \frac{z}{2}.$$

Answers:

1. $8x - 8y - z = 4$, and
$$\frac{x-2}{8} + \frac{y-1}{-8} + \frac{z-4}{-1}.$$
2. $x + y - z = 1$ and
$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{-1}.$$
3. $z = -1$ is the tangent plane while $x = 1,\ y = 1,\ z = -1 + t$ is the normal line.
4. $x - y + 2z - \frac{\pi}{2} = 0$, and
$$\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z - \frac{\pi}{2}}{2}$$
5. $17x + 11y + 5z = 60$ and
$$\frac{x-3}{17} = \frac{y-4}{11} = \frac{z+7}{5}.$$
6. $x + 11y + 5z = 18$ and
$$\frac{x-1}{1} = \frac{y-2}{11} = \frac{z+1}{5}.$$
7. $2x + y + 11z = 25$ and
$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{11}.$$
8. $5x + 4y + z = 28$ and
$$\frac{x-2}{5} = \frac{y-3}{4} = \frac{z-6}{1}.$$
9. The tangent plane is parallel to the $xy-$plane at the points $(0, 3, 3)$ and $(0, 3, -7)$. It is parallel to the $yz-$plane at the points $(5, 3, -2)$ and $(-5, 3, -2)$. It is parallel to the $xz-$plane at the points $(0, -2, -2)$ and $(0, 8, -2)$.
10. $4x - 2y - 3z = 3$.