Exercises: Space Curves and Surfaces

Exercise Set 1

Find the equation of the tangent line and the normal plane to the curves defined by the following vector functions:

1. $\mathbf{r}(t) = \left(\frac{t^4}{4}, \frac{t^3}{3}, \frac{t^2}{2}\right)$ at any value of t.

2. $\mathbf{r}(t) = \left(at, \frac{at^2}{2}, \frac{at^3}{3}\right)$ at the value of t corresponding to $P(6a, 18a, 72a)$ where a is a given constant.

3. $\mathbf{r}(t) = \left(t - \sin t, 1 - \cos t, 4 \sin \frac{t}{2}\right)$ at the value of t corresponding to $P(\frac{\pi}{2} - 1, 1, 2\sqrt{2})$

4. $\mathbf{r}(t) = \left(e^t, e^{-t}, t\sqrt{2}\right)$ at the value of t corresponding to $P(e, e^{-1}, \sqrt{2})$

5. $\mathbf{r}(s) = (\sin s, \cos s, \tan s)$ at the value of s corresponding to $P(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1)$

6. $\mathbf{r}(t) = \left(t^2, 1 - t, t^3\right)$ at the value of t corresponding to $P(1, 0, 1)$

7. $\mathbf{r}(t) = \left(t^3 - t^2 - 5, 3t^2 + 1, 2t^3 - 16\right)$ at the point P where $t = 2$.

Find the equation of the tangent line and the normal plane to the curve defined by the intersection of the following surfaces:

8. The curve defined by the intersection of the surfaces $x^2 + y^2 = 10$ and $y^2 + z^2 = 25$ at $P(1, 3, 4)$.

9. The curve defined by the intersection of the surfaces $x^2 + y^2 = z^2$ and $x = y$ at $P(x_0, y_0, z_0)$.

10. The curve defined by the intersection of the surfaces $y^2 = x$ and $x^2 = z$ at $P(1, 1, 1)$.

Answers:

1. $\dfrac{x - t^4/4}{t^2} = \dfrac{y - t^3/3}{t} = \dfrac{z - t^2/2}{1}$, and
$$t^2 x + ty + z = \frac{t^6}{4} + \frac{t^4}{3} + \frac{t^2}{2}.$$

2. $\dfrac{x - 6a}{1} = \dfrac{y - 18a}{6} = \dfrac{z - 72a}{36}$, and
$$x + 6y + 36z = 2706a.$$

3. $\dfrac{x - \frac{\pi}{2} + 1}{1} = \dfrac{y - 1}{1} = \dfrac{z - 2\sqrt{2}}{\sqrt{2}}$, and
$$x + y + z\sqrt{2} = \frac{\pi}{2} + 4.$$

4. $\dfrac{x - e}{e} = \dfrac{y - 1/e}{-1/e} = \dfrac{z - \sqrt{2}}{\sqrt{2}}$, and
$$ex - \frac{1}{e}y + \sqrt{2}z = e^2 - \frac{1}{e^2} + 2.$$

5. $\dfrac{x - \sqrt{2}/2}{\sqrt{2}} = \dfrac{y - \sqrt{2}/2}{-\sqrt{2}} = \dfrac{z - 1}{4}$, and
$$x\sqrt{2} - y\sqrt{2} + 4z = 4.$$

6. $\dfrac{x - 1}{2} = \dfrac{y}{-1} = \dfrac{z - 1}{3}$, and
$$2x - y + 3z = 5.$$

7. $\dfrac{x + 1}{2} = \dfrac{y - 13}{3} = \dfrac{z}{6}$, and
$$2x + 3y + 6z = 37.$$

8. $\dfrac{x - 1}{12} = \dfrac{y - 3}{-4} = \dfrac{z - 4}{3}$, and
$$12x - 4y + 3z = 12.$$

9. $\dfrac{x - x_0}{z_0} = \dfrac{y - y_0}{z_0} = \dfrac{z - z_0}{x_0 + y_0}$, and
$$\frac{1}{x_0 + y_0}x + \frac{1}{x_0 + y_0}y + \frac{z}{z_0} = 2.$$

10. $\dfrac{x - 1}{1} = \dfrac{y - 1}{1/2} = \dfrac{z - 1}{2}$, and
$$2x + y + 4z = 7.$$