

Print Name :

SOLUTIONS

Student Number:

PART I: Multiple Choice Questions

50 minutes

Choose and CIRCLE only ONE answer

Non-programmable calculators permitted

One blank sheet for rough work permitted but DO NOT HAND IN.

1. [2 marks] Let $f(x) = \text{Arctan}(2x)$. What is the value of $f'(1)$?

- (a) $\frac{2}{5}$, (b) 2, (c) $\frac{1}{2}$, (d) 0.

2. [2 marks] Evaluate the limit: $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$.

- (a) The limit does not exist, (b) 1, (c) $\frac{2}{3}$, (d) $\frac{3}{2}$.

3. [2 marks] Let $f(x) = \text{Arctan } x + \text{Arccos}(x^2)$. Then $f(1)$ is equal to:

- (a) $\frac{\pi}{2}$, (b) $\frac{\pi}{4}$, (c) 0, (d) $\frac{\pi}{3}$.

4. [2 marks] A function f has the property that $f(1) = 6$ and $f'(1) = 2$. Assuming that f has an inverse function, F , calculate the value of the derivative of F at $x = 6$, that is, $F'(6)$:

- (a) 2, (b) +1, (c) $\frac{1}{2}$, (d) 0.

5. [2 marks] Answer TRUE or FALSE:

The function f defined by $f(x) = \text{Arcsin}(x - 2)$ is differentiable at $x = 2$.

- (a) TRUE, (b) FALSE

PART II: Show all work here.

No additional pages will be accepted

6. [5+5 marks] Find the required limits:

a) $\lim_{x \rightarrow 0^+} \frac{\sin(3x)}{5x}$

use L'Hopital's. $\lim_{x \rightarrow 0} \frac{3 \cos 3x}{5} = \frac{3}{5}$

b) $\lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x})$

took at $\sqrt{x}(\sqrt{x+1} - \sqrt{x})$

- - a) Answer = $\frac{3}{5}$.

$$= \frac{\sqrt{x}(\sqrt{x+1} + \sqrt{x})(\sqrt{x+1} - \sqrt{x})}{\sqrt{x+1} + \sqrt{x}} = \sqrt{x} \frac{(x+1-x)}{\sqrt{x+1} + \sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x+1} + \sqrt{x}} \rightarrow$$

7. [5+5 marks]

a) $f(x) = \text{Arccos}(\sin(x^3))$. Find $f'(0)$.

b) Calculate the approximate value of the root of the function

$$f(x) = \frac{x}{2} - \sin x$$

near $x_0 = \frac{\pi}{2}$ using the iterations x_0, x_1, x_2 in Newton's method.

a) $f(x) = \text{Arccos } \square$.

$$f'(x) = -\frac{1}{\sqrt{1-\square^2}} \Delta \square, \quad \square = \sin(x^3).$$

$$\Delta \square = 3x^2 \cos x^3.$$

$$\therefore f'(0) = -\frac{1}{\sqrt{1-(\sin 0)^3}} \cdot (3x^2 \cos x^3) \Big|_{x=0}.$$

$$\boxed{-f'(0) \equiv 0.}$$

(7b) ignore)

→ 6b.) Factor $\sqrt{x} \Rightarrow$

$$\frac{\sqrt{x}}{\sqrt{x} + \sqrt{x+1}} = \frac{\sqrt{x}}{\sqrt{x}(1 + \sqrt{\frac{x+1}{x}})} = \frac{1}{1 + \sqrt{\frac{x+1}{x}}} = \frac{1}{1 + \sqrt{1 + \frac{1}{x}}}$$

$$= \frac{1}{1 + \sqrt{1 + \frac{1}{x}}} \Rightarrow \lim_{x \rightarrow \infty} \left(\frac{1}{1 + \sqrt{1 + \frac{1}{x}}} \right) = \frac{1}{1+1} = \left(\frac{1}{2} \right)$$