numbers having almost 188 million digits of precision. It is highly unlikely that a solution set of Fermat's Last Theorem will ever be discovered with the use of a computer with such huge number representations (storage) and with such high order precision (computer time) required before adequate testing can be performed.

References

1. Abstract, Bull. Amer. Math. Soc., 20 (1913) 68–69.
2. Selfridge and Pollock, Notices Amer. Math. Soc., January 1964, p. 97.

NOTE ON NON-EUCLIDEAN PRINCIPAL IDEAL DOMAINS

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It is well known that every Euclidean domain is a principal ideal domain. In [**4**] Wilson's object is to show, in a manner accessible to students in an undergraduate algebra class, that the ring of integers of the field $Q(\sqrt{-19})$ is a principal ideal domain which is not Euclidean. The proof that it is a principal ideal domain is based on a theorem of Dedekind and Hasse and the proof that it is not Euclidean is based upon the work of Motzkin [**2**]. However, as much of what Wilson does in the latter proof is unnecessary for the required purpose, it is our purpose to give a simpler treatment.

If D is an integral domain, we let $\tilde{D}$ denote the collection of units of D together with 0, so that $D - \tilde{D} = \emptyset$ if and only if D is a field. An element $u \in D - \tilde{D}$ is called a universal side divisor if for any $x \in D$ there exists some $z \in \tilde{D}$ such that $u \mid x - z$.

THEOREM. *Let D be an integral domain which is not a field (so that $D - \tilde{D} \neq \emptyset$) and which has no universal side divisors. Then D is not Euclidean.*

Proof. Suppose that D is a Euclidean domain, with Euclidean function d, which has no universal side divisors. Consider the nonempty subset $S = \{d(v) : v \in D - \tilde{D}\}$ of the nonnegative integers. It possesses a least element, say $d(u)$, $u \in D - \tilde{D}$. For any $x \in D$ there exists $y, z \in D$ such that $x = uy + z$, where either (i) $z = 0$ or (ii) $z \neq 0$ and $d(z) < d(u)$. If (i) holds then $u \mid x$. If (ii) holds, by the minimality of $d(u)$, z must be a unit. Thus in both cases $u \mid x - z$ for some $z \in \tilde{D}$, and so u is a universal side divisor which is impossible.

COROLLARY. *The rings of integers of $Q(\sqrt{-19})$, $Q(\sqrt{-43})$, $Q(\sqrt{-67})$, $Q(\sqrt{-163})$ are not Euclidean.*

Proof. Let $D = 19, 43, 67$ or 163 and suppose that the ring

$$R = \{a + b(1 + \sqrt{-D})/2 : \ a, b \text{ integers}\}$$

of integers of $Q(\sqrt{-D})$ contains a universal side divisor u. As the only units of R are ± 1, u must be a nonunit divisor of 2 or 3. In R, 2 and 3 are irreducible and therefore the only possible universal side divisors are 2, -2, 3, and -3.

However, none of these divides any of the integers

$$\tfrac{1}{2}(1+\sqrt{-D}),\ \tfrac{1}{2}(3+\sqrt{-D}),\ \tfrac{1}{2}(-1+\sqrt{-D}),$$

so that no such universal side divisor u can exist. Hence by the theorem, R is not Euclidean.

Recently Stark [3] has shown that the only complex quadratic fields $Q(\sqrt{-D})$ whose rings of integers are principal ideal domains are given by $D = 1, 2, 3, 7, 11, 19, 43, 67, 163$ and since it is well known [1] that the first five of these are Euclidean (with respect to the norm), the above corollary gives all the complex quadratic fields whose rings of integers are non-Euclidean principal ideal domains.

References

1. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, 4th ed., Oxford, New York, 1962, Theorem 246, p. 213.

2. Th. Motzkin, The Euclidean algorithm, Bull. Amer. Math. Soc., 55 (1949) 1142–1146.

3. H. M. Stark, A complete determination of the complex quadratic fields of class-number one, Michigan Math. J., 14 (1967) 1–27.

4. J. C. Wilson, A principal ideal ring that is not a Euclidean ring, this MAGAZINE, 46 (1973) 34–38.

NOTES AND COMMENTS

David Kullman notes that the article *5-con triangles* by Richard G. Pawley in the Mathematics Teacher, May 1967, vol. 60, pp. 438–443, includes results equivalent to several of those in the article *Almost congruent triangles* by Bruce B. Peterson in our September 1974 issue.

From Aaron R. Todd regarding *Almost congruent triangles*, this MAGAZINE, vol. 47, Sept. 1974, No. 4 by Robert T. Jones and Bruce Peterson: The authors should mention their use of continuity in at least one of the several places they need the concept, especially as they ban it so forcibly in their proof of the existence of almost congruent triangles. For that matter, a principle of continuity such as the following is useful in filling a gap in traditional constructions of triangles: *If a point on the arc of a circle lies on one side of a straight line and another point of the arc lies on the other side of the line, then there is a point of the arc lying on the line.*

Comment by Graham Lord, Temple University, Philadelphia, Pa. on *A note on Mersenne numbers* by Steve Ligh and Larry Neal, this MAGAZINE, vol. 47, September 1974, No. 4, pp. 231–233. Theorem 1 of the note is just a special case of the result: If n is a natural number > 1, then $2^n - 1$ is not the mth power of a natural number $m > 1$. This result is quoted in *Elementary theory of numbers* by W. Sierpinski who gives as a reference C. G. Gerono, Nouv. Ann. Math. (2) 9(1870) pp. 469–471, 10 (1871) pp. 204–206.