Canad. Math. Bull. 11 (1968), 145.

ADDENDUM TO "ON A THEOREM OF NIVEN"

Kenneth S. Williams

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Gordon Pall has kindly pointed out that the result of this paper was obtained by him in

Sums of two squares in a quadratic field, Duke Math. Jour., 18 (1951), 399-409.

He gives the result in a slightly different form on page 405. We note that the two formulae are indeed the same. In my paper write (3) as

$$z = \epsilon(1+i)^{\alpha} \pi_1^{\alpha_1} \cdots \pi_s^{\alpha_s} \bar{\pi}_1^{\alpha_1'} \cdots \bar{\pi}_s^{\alpha_s'} q_1^{\beta_1} \cdots q_\ell^{\beta_\ell}$$

so that

$$z\bar{z} = 2^{\alpha} p_1^{\alpha_1+\alpha_1'} \cdots p_s^{\alpha_s+\alpha_s'} q_1^{2\beta_1} \cdots q_\ell^{2\beta_\ell} .$$

If $p_i^{\gamma_i} || (x, y)$, then in Pall's notation $\gamma_i = \min(\alpha_i, \alpha_i')$

and
$$\alpha_i + \alpha_i' = 2\gamma_i + \delta_i ,$$

giving $\gamma_i + \delta_i = \alpha_i + \alpha_i' - \min(\alpha_i, \alpha_i') = \max(\alpha_i, \alpha_i')$.

Hence
$$\begin{aligned}(1+\gamma_i)(1+\gamma_i+\delta_i) &= (1+\min(\alpha_i, \alpha_i'))(1+\max(\alpha_i, \alpha_i')) \\ &= (1+\alpha_i)(1+\alpha_i') ,\end{aligned}$$

showing that Pall's formula (22) is the same as my formula (19).