Math. Gaz. 48 (1964), 213.

3094. On the factorization of the first four composite Mersenne numbers

The Mersenne Numbers M_p are defined by $2^p - 1$ where p is prime. I give simple factorizations of the first four composite ones.

$$\begin{aligned} M_{11} &= 32 \cdot 2^6 - 1 \\ &= 33 \cdot 2^6 - 2^6 - 1 \\ &= 3 \cdot 11 \cdot 2^6 - 2^3 \cdot 11 + 2^3 \cdot 3 - 1 \\ &= (3 \cdot 2^3 - 1)(11 \cdot 2^3 + 1) \\ &= 23 \cdot 89 \end{aligned}$$

Cataldi 1588

$$\begin{aligned} M_{23} &= 128 \cdot 2^{16} - 1 \\ &= 81 \cdot 2^{16} + 47 \cdot 2^{16} - 1 \\ &= 48^4 - 1 + 47 \cdot 2^{16} \\ &= 47 \cdot 178{,}481 \end{aligned}$$

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$$\begin{aligned} M_{29} &= 2{,}097{,}152 \cdot 2^8 - 1 \\ &= (1103 \cdot 1897 + 69^2) \cdot 2^8 - 1 \\ &= 1103 \cdot 1897 \cdot 2^8 + 1104^2 - 1 \\ &= 1103 \cdot (1897 \cdot 2^8 + 1105) \\ &= 1103 \cdot (1748 \cdot 2^8 + 149 \cdot 2^8 + 41309 - 1748 \cdot 23) \\ &= 1103 \cdot (1748 \cdot 233 + 79453) \\ &= 1103 \cdot (1748 \cdot 233 + 341 \cdot 233) \\ &= 1103 \cdot 2089 \cdot 233 \end{aligned}$$

Euler 1750

$$\begin{aligned} M_{37} &= 4096 \cdot 2^{25} - 1 \\ &= 16807 \cdot 2^{25} - 12711 \cdot 2^{25} - 1 \\ &= 7^5 \cdot 2^{25} - 57 \cdot 223 \cdot 2^{25} - 1 \\ &= 224^5 - 1 - 57 \cdot 223 \cdot 2^{25} \\ &= 223 \cdot 616{,}318{,}177 \end{aligned}$$

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