EXERCISES 2, QUESTION 7

7. Use Theorem 2.3.1 to show that $\mathbb{Z} + \mathbb{Z}\sqrt{26}$ is not Euclidean with respect to ϕ_{26}.

Solution. Let

$$m = 26,\ p = 3,\ q = 7,\ t = 4,\ u = 2,\ r = 8.$$

Then

$$\left(\frac{m}{p}\right) = \left(\frac{26}{3}\right) = \left(\frac{-1}{3}\right) = -1,$$
$$\left(\frac{m}{q}\right) = \left(\frac{26}{7}\right) = \left(\frac{-2}{7}\right) = -1,$$

$$pt + qu = 3 \times 4 + 7 \times 2 = 12 + 14 = 26 = m,\ p \nmid t,\ q \nmid u,$$
$$r^2 \equiv 64 \equiv 12 \equiv pt \pmod{26}.$$

Hence, by Theorem 2.3.1, $\mathbb{Z} + \mathbb{Z}\sqrt{26}$ is not Euclidean with respect to ϕ_{26}. ∎

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