

CHAPTER 1, QUESTION 23

23. Prove that $17 - 3\sqrt{3} \sim 83 + 47\sqrt{3}$ in $\mathbb{Z} + \mathbb{Z}\sqrt{3}$.

Solution. We have

$$\frac{83 + 47\sqrt{3}}{17 - 3\sqrt{3}} = \frac{(83 + 47\sqrt{3})(17 + 3\sqrt{3})}{(17 - 3\sqrt{3})(17 + 3\sqrt{3})} = \frac{1834 + 1048\sqrt{3}}{262} = 7 + 4\sqrt{3}.$$

As

$$7^2 - 3 \cdot 4^2 = 49 - 48 = 1,$$

we see by Question 17 that

$$7 + 4\sqrt{3} \in U(\mathbb{Z} + \mathbb{Z}\sqrt{3}).$$

Hence

$$83 + 47\sqrt{3} \sim 17 - 3\sqrt{3}. \quad \blacksquare$$

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