

CHAPTER 1, QUESTION 19

19. Prove that $\langle 1 - 3i, 3 - i \rangle$ is a principal ideal in $\mathbb{Z} + \mathbb{Z}i$ by finding a generator for this ideal.

Solution. We have

$$\begin{aligned}\langle 1 - 3i, 3 - i \rangle &= \langle 1 - 3i, i(3 - i) \rangle \quad (\text{as } i \text{ is a unit}) \\ &= \langle 1 - 3i, 1 + 3i \rangle \\ &= \langle 1 - 3i + 1 + 3i, 1 + 3i \rangle \\ &= \langle 2, 1 + 3i \rangle \\ &= \langle 2, 1 + 3i - 2i \rangle \\ &= \langle 2, 1 + i \rangle \\ &= \langle (1 + i)(1 - i), 1 + i \rangle \\ &= \langle 1 + i \rangle \langle 1 - i, 1 \rangle \\ &= \langle 1 + i \rangle\end{aligned}$$

by Question 7. ■

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