Research Description

Minyi Huang Feb. 2006

(1) Distributed optimization in multi-agent stochastic systems:

Introduction and Motivation

There has been a rapidly increasing research interest in the analysis and optimization of large-scale dynamical systems involving multiple agents, which arise in a variety of areas including communication networks, economics and social science, biological systems, among others. The great complexity of these systems – in the form of high dimensionality, uncertainty, and complicated interaction among different constituent parts – results in significant limitation in human understanding in the behavior of their agents and that of the overall system, and this hinders further efforts in optimization of system performance as measured by various means. Until nowadays, large-scale system modelling and optimization remains an area with major challenging problems.

In this research we are particularly interested in an important class of large-scale stochastic systems where each agent interacts with all other agents via coupling in their individual dynamics and costs. A remarkable feature is that for a given agent, any other single agent only produces a negligible impact while the overall population's effect on it is significant. This kind of individual-mass interacting systems, taking their specific forms, arise in different areas ranging from (oligopoly) economic models where each agent competes with the mass of all others over a market, network resource allocation such as power control in cellular networks and Internet traffic control, to biological systems where individual animals compete for food and reproduction opportunities [18, 7, 6, 1, 14, 21, 19]. A systematic investigation of systems with such individual-mass interactions will provide useful insights for understanding many complex phenomena related to human activities, engineering applications and the natural environment, and may extend the application horizon of systems and control theory.

Fundamental Limitation of Traditional Approaches

In view of the self-interest seeking behavior of the agents, it is natural to approach the optimization problem in a game theoretic framework [3, 8]. Specifically, under the stochastic dynamic evolution of individual agent's state, one might attempt to find a solution via existing noncooperative dynamic game theory. However, under large population conditions, this leads to prohibitively high complexity for analysis and computation, even for linear models. Furthermore, under this optimization framework, the resulting solution, if computable at all, is centralized and unrealistic for implementation since each agent needs to know the states of all other agents. The limitation of the existing approach motivates us for the search of an efficient optimization paradigm.

The New Paradigm for Large Population Stochastic Dynamic Games:

——————— Breaking Curse of Dimensionality, and Research Advance

Starting with a linear-quadratic-Gaussian (LQG) game model with cost coupling, my Ph.D. thesis work initiated a general methodology via state aggregation for analyzing the individual and mass behavior in large-scale systems within a decentralized game theoretic framework. In this individual-mass interacting model, utilizing the full system state for a given individual's strategy optimization is not only a very demanding task, but also unlikely to generate considerable additional benefit compared to an appropriately constructed decentralized strategy.

Thus, we take a fundamentally different approach based on characterizing an individual-mass consistency relationship within the large-population limit. By doing this we only need to introduce two objects including a "representative" agent and the mass effect, which forms the basis for overcoming

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the dimensionality difficulty. The crucial step here is to use state aggregation to extract the population influence into a deterministic trajectory (mass effect) against which each agent optimizes its strategy. The individual strategies are determined so that they are each an optimal response to the underlying mass effect and they also collectively produce the same mass effect. We call this design scheme the *Nash Certainty Equivalence Principle*, which leads to *decentralized* ε-Nash strategies where the offset ε goes to zero as the population size increases. Our analysis reveals under mild conditions a stable population behavior resulting from localized self optimizing agents.

In a series of papers [15, 13, 12, 11, 10, 9], this methodology is substantially extended to more general models with features such as both cost and dynamic coupling, nonuniform agents, noisy state measurements, and certain nonlinearities. Remarkably, our recent work discovered an intimate connection between our optimization approach and statistical mechanics for interacting particle systems, as illuminated by the celebrated McKean-Vlasov equation and its controlled version [9, 5, 2, 4, 17, 16, 20, 22]. As the basis for this connection, the models in these areas share the common feature — the system behavior at the macroscopic level is largely governed by the interaction between an individual and the mass of all others.

The Nash certainty equivalence based methodology may be extended to more general nonlinear stochastic systems. Our initial analysis indicates that the overall distributed optimization problem may be decomposed into a standard low dimensional stochastic control problem to be solved along an "optimal" population distribution process and a Fokker-Planck equation for the population distribution evolution [9]. As an analytic tool, the Fokker-Planck equation has a dual role for describing the population behavior and also the statistical property of a "representative" agent. The subsequent analysis relies on combining the state aggregation procedure, stochastic analysis and fixed point techniques. In the end, this leads to the construction of localized control strategies for the agents.

This research is of importance for dealing with complexity in dynamic optimization of large-scale systems and has methodological implications in many complex systems arising in the socio-economic and engineering areas and evolutionary biology. The generalization of our methodology to other modelling situations reflecting practical system features will be investigated in the future.

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